Homework Problems - Math 101

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Sets

1. True or false.
   a. $2 \in \{1,2,3,5,7\}$
   b. $3 \in \{2,10,a,w,0\}$
   c. $b \in \{h,i,j,k\}$
   d. $\textbf{good} \in \{\textbf{happy},\textbf{sad},\textbf{good},\textbf{bad}\}$
   e. $4 \in \{4\}$
   f. $4 \in 4$
   g. $\{4\} \in \{1,2,3,4\}$
   h. $\{4\} \in \{\{1\},\{2\},\{3\},\{4\}\}$
   i. $\{4\} \in \{\{\{4\}\},4\}$
   j. $\{1,\{2\}\} \in \{1,2,\{1,\{2\}\},\{1,\{2\},\{1,\{2\}\}\}\}$
2. For each pair of sets, determine the union of the two sets. Do not list elements more than once in a given set.
   a. $\{1,2,3\} \cup \{4,5,6\}$
   b. $\{a,2,x\} \cup \{6,w\}$
   c. $\{1,2,3\} \cup \{2,3,4\}$
   d. $\{a,b,c\} \cup \{c,a,b\}$
   e. $\{1,2,3,4\} \cup \{\quad\}$
   f. $\{\quad\} \cup \{\quad\}$
   g. $\{\textbf{sad},\textbf{good},\textbf{fun}\} \cup \{\textbf{happy},\textbf{sad},\textbf{good},\textbf{bad}\}$
   h. $\{1,\{2\},\{3\}\} \cup \{2,\{3\}\}$
   i. $\{\{a,\{x\}\},\{3\}\} \cup \{a,\{x,3\}\}$

Iteration

1. Consider the following game. Starting with a positive whole number, if the number is odd multiply it by three and then add one, but if it is even divide it by two. Then iterate this process so that the answer you got (the output of the process) becomes the new starting number (the input).

   For example, if we start with the number 3, then since 3 is odd we multiply it by 3 and add 1 to get 10. Since 10 is the output from our first iteration of this process, it becomes the input for the next iteration. 10 is even so we divide it by two to get 5 as our new output number. 5 is odd so we multiply it by three and add 1 to get 16. 16 is even so we divide it by two to get 8. Since 8 is even we divide it by two to get 4, and so on. We can list the numbers we get by iterating this process starting with 3 in order as follows.

   $$3,10,5,16,8,4,2,1,4,2,1,4,2,1,\ldots$$

   Notice that after five iterations the numbers form a repeating pattern of 4 then 2 then 1 then back to 4 again, and the process is “trapped” in this loop forever.

   Let’s consider what happens when we start with numbers other than three:

$1, 4, 2, 1, 4, 2, 1\ldots$

$2, 1, 4, 2, 1, 4, 2, 1, \ldots$

$3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots$

$4, 2, 1, 4, 2, 1, \ldots$

$5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots$

$6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots$

$7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots$

From this table we see that for every starting number from 1 to 7, the process eventually gets trapped in the $4, 2, 1$ cycle.

Extend the table above for the starting numbers 8 through 26. Continue putting entries in each row until the process is trapped in the $4, 2, 1$ cycle.

2. Consider the following iterative process. On a standard scientific calculator, input any positive number you like, and then press the square root button. The number you obtain is the output of this process. Now iterate the process many time (i.e. on most calculators this means you just keep pressing the square root button). What happens? Try several different starting numbers and record your results in a table. Describe your results. Can you make any predictions about the behavior of iterating this system?

3. Repeat the previous problem for some other buttons on the calculator (cos, sin, ln, 1/x, etc. not +,-,multiplication or division). You can choose any two buttons you like. Record your results in a table and describe your results. Can you make any predictions about the behavior of iterating these processes?

Discrete Dynamical Systems

1. Given functions f, g, h whose rules are $f(x) = 2x + 3,\ g(x) = 1 - x^2,\ h(x) = x(2 - x)$ compute each of the following. You may use a calculator if you need to.

- **a.** $f(0)$
- **b.** $g(0)$
- **c.** $h(2)$
- **d.** $h(0)$
- **e.** $g\left(\frac{1}{2}\right)$
- **f.** $f\left(-\frac{5}{2}\right)$
- **g.** $f(-2)$
- **h.** $g(-1.35)$
- **i.** $h(0.257)$
- **j.** $g(f(3))$
- **k.** $h(h(1))$
- **l.** $h(g(f(1)))$
- **m.** $f(f(f(3)))$
- **n.** $h(a)$
- **o.** $g(a + b)$
- **p.** $f(1 + b)$

2. Compute the first six iterations of the given function starting with the given seed, x_0.

- **a.** $f(x) = 2x,\ x_0 = 1$

b. $g(x) = 2 + x$, $x_0 = 1$
c. $h(x) = -x$, $x_0 = -2$
d. $h(x) = -x$, $x_0 = 2$
e. $L(x) = x(1 - x)$, $x_0 = 0.6$
f. $S(x) = x^2 - 2$, $x_0 = -0.4$
g. $t(x) = \frac{1}{x}$, $x_0 = \frac{1}{7}$
h. $t(x) = \frac{1}{2}x$, $x_0 = 3$
i. $r(x) = x^2 - \frac{3}{4}$, $x_0 = \frac{3}{2}$
j. $g(x) = x^2 - \frac{21}{16}$, $x_0 = \frac{1}{4}$

3. Classify each of the orbits in the previous problem as cyclic, eventually cyclic, or neither. If the orbit is cyclic, say what kind of cycle it is (i.e. fixed point, 2-cycle, 3-cycle, etc.).

4. Classify each of the seeds starting numbers x_0 in the previous problem as cyclic, eventually cyclic, or neither. If the seed is cyclic, find its minimum period.

5. Find a fixed point for the function $f(x) = -x$.

6. Consider the function $g(x) = 1 - x$.
a. Find a three different 2-cycles for g.
b. Find a fixed point for g.
c. Compute $g(g(x))$. What does this indicate about the orbits of numbers for g?

7. For each of the following, find a rule for a function that can produce the given orbit.
a. $4, 5, 6, 7, 8, 9, 10, \ldots$
b. $2, 5, 8, 11, 14, 17, 20, \ldots$
c. $10, 8, 6, 4, 2, 0, -2, -4, \ldots$
d. $3, -6, 12, -24, 48, -96, \ldots$
e. $\overline{-2, 2}$
f. $\overline{1}$
g. $\overline{5, \frac{1}{5}}$
h. $\frac{1}{3}, \frac{1}{9}, \frac{1}{81}, \frac{1}{6561}, \ldots$
i. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

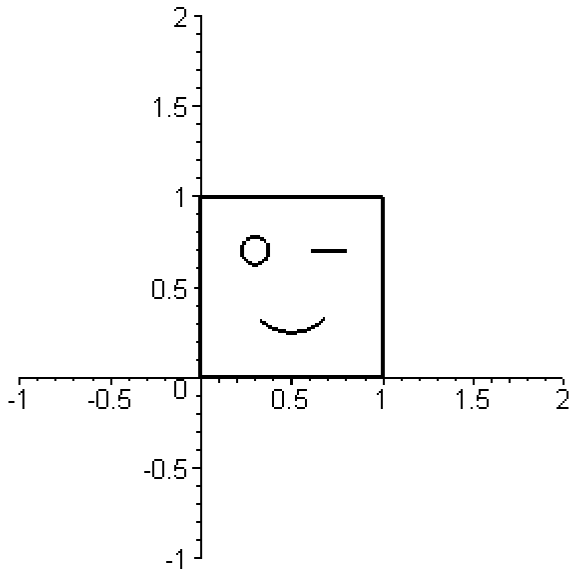
HeeBie Geebies

1. Use a colored highlighter marker and the 27×27 graph paper (available at the web site) to draw the third iteration of the indicated Grid Based Fractal.
a. $\text{GB}(3; 7)$
b. $\text{GB}(3; 2, 8)$
c. $\text{GB}(3; 2, 4, 6, 8)$
d. $\text{GB}(3; 1, 3, 7, 9)$

2. Use a colored highlighter marker and the appropriate sized graph paper (available at the web site) to draw the indicated iteration of the following HeeBGB Fractals.
a. $\text{HeeBGB}(Up, Up, Dn, none)$, 5^{th} iteration, use 32×32 graph paper.
b. $\text{HeeBGB}(Lt, Lt, Lt, none)$, 5^{th} iteration, use 32×32 graph paper.
c. $\text{HeeBGB}(Up, Rt, Lt, none)$, 5^{th} iteration, use 32×32 graph paper.
d. $\text{HeeBGB}(Up, none, none, Dn, Dn, none, Up, Dn, Up)$, 3^{rd} iteration, use 27×27 graph paper.

Affine Transformations

In each of the following problems, "Mr. Face" refers to the following figure:



1. Draw the image of Mr. Face under each of the following affine transformations. Use graph paper and be sure to label the axes. Draw each image on a separate graph, don't draw all of them on a single graph like you do for an IFS, i.e. each transformation is a separate question.

a.

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	3	0	0	0	0

b.

	r	s	θ	ϕ	e	f
T_0	1	1	−90	−90	0	0

c.

	r	s	θ	ϕ	e	f
T_0	1	1	0	0	2	−1

d.

	r	s	θ	ϕ	e	f
T_0	1	−1	0	0	0	0

e.

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	$\frac{1}{2}$	90	90	$\frac{1}{2}$	$\frac{1}{2}$

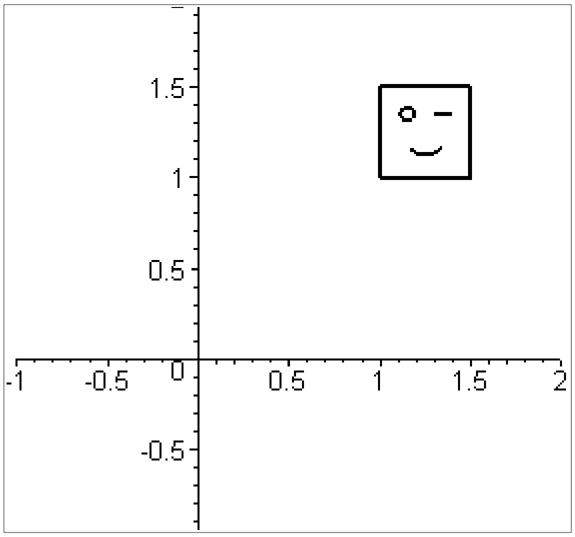
f.

	r	s	θ	ϕ	e	f
T_0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	−1	0

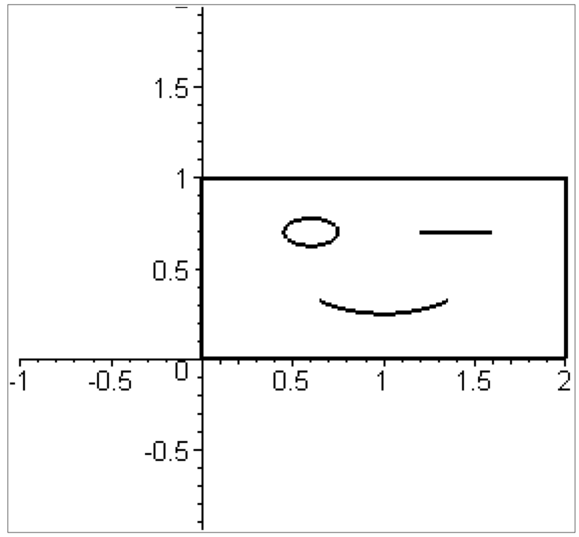
g.

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	−1	−45	−45	0	−1

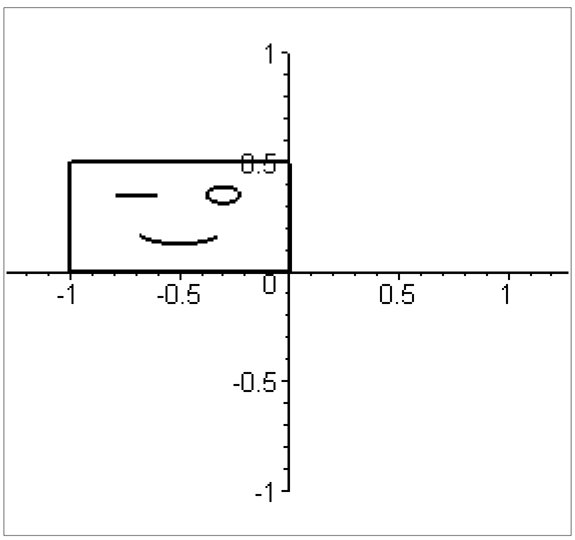
2. For each of the following images of Mr. Face, find the values r, s, θ, ϕ, e, f for an affine transformation that will send him there from his original position.


1.5
1
0.5
0
-1
-0.5
0.5
1
1.5
2
-0.5

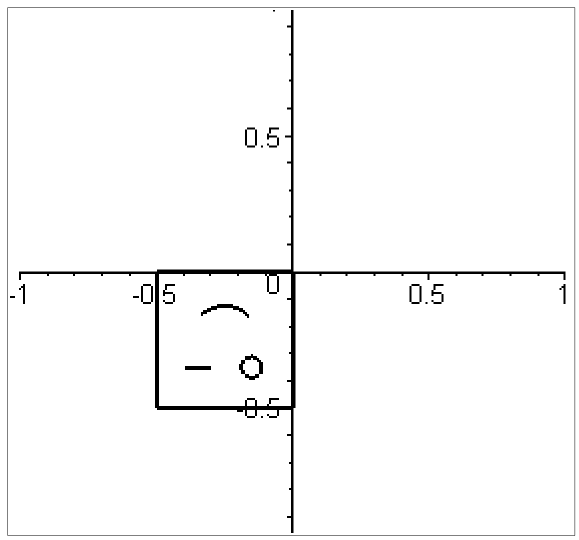
a.


1.5
1
0.5
0
-1
-0.5
0.5
1
1.5
2
-0.5

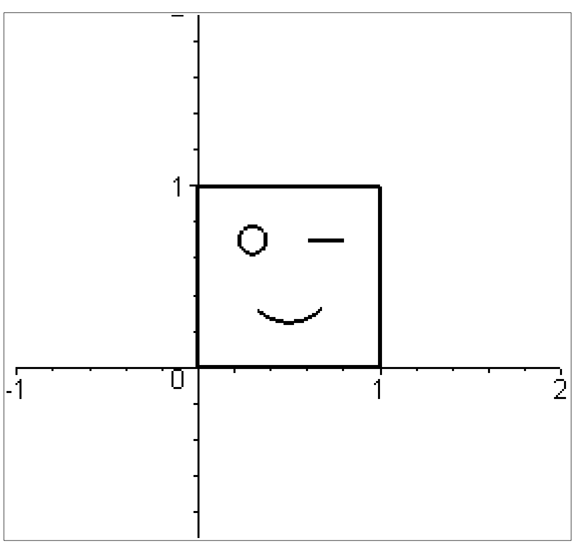
b.


1
0.5
-1
-0.5
0
0.5
1
-0.5
-1

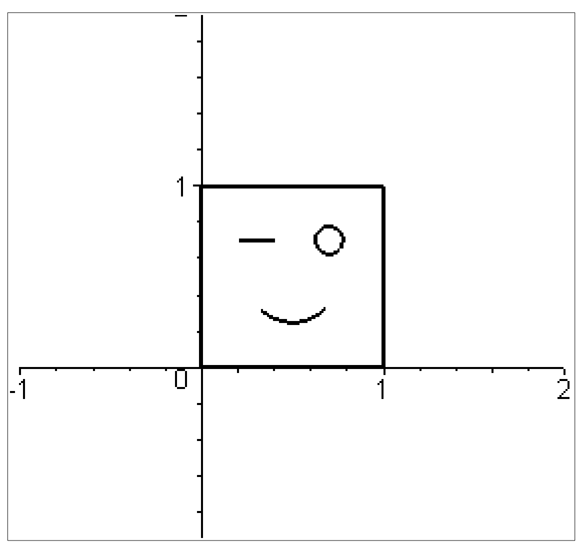
c.


0.5
-1
-0.5
0
0.5
1
-0.5

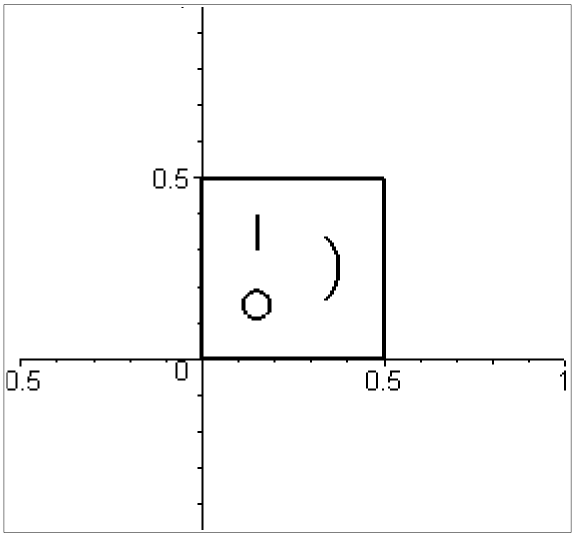
d.


1
-1
0
1
2

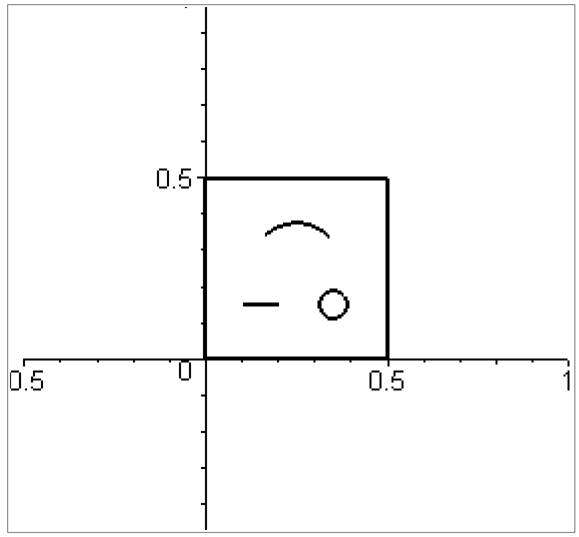
e.


1
-1
0
1
2

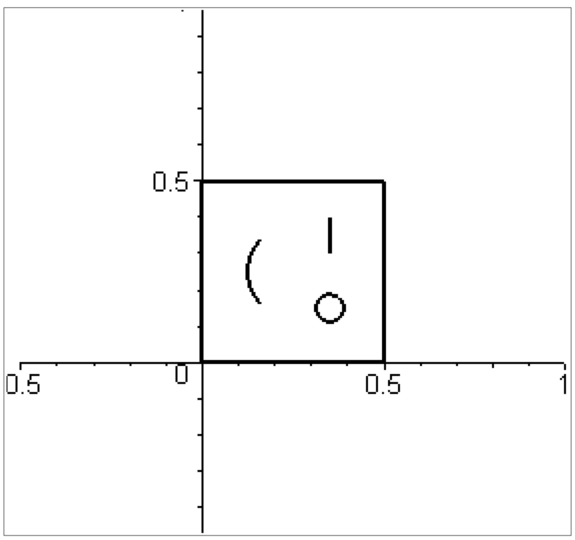
f.



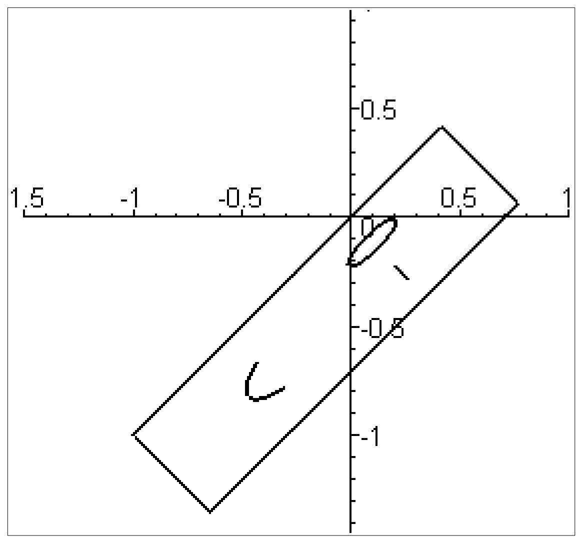
g.



h.



i.



j. (Hint: use a ruler)

IFS

1. For each of the following IFS's, draw the first three iterations starting with Mr. Face using the deterministic method. [Note: Use the 8x8 graph paper for each iteration (from the web site).]

a.

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
T_1	$\frac{1}{2}$	$\frac{1}{2}$	-90	-90	$\frac{1}{2}$	$\frac{1}{2}$
T_2	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$

b.

	r	s	θ	ϕ	e	f
T_0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	1
T_1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
T_2	$\frac{1}{2}$	$\frac{1}{2}$	90	90	1	0

c.

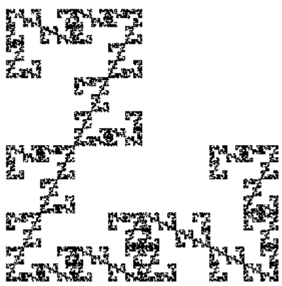
	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$
T_1	$\frac{1}{2}$	$\frac{1}{2}$	–90	–90	0	$\frac{1}{2}$
T_2	$\frac{1}{2}$	$\frac{1}{2}$	90	90	1	0

d.

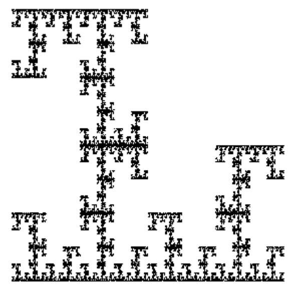
	r	s	θ	ϕ	e	f
T_0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0
T_1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	1	0
T_2	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	1

2. Match each of the following fractals with the IFS in the previous problem that produced it.

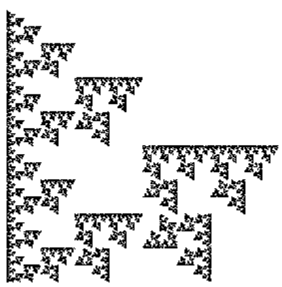
a.



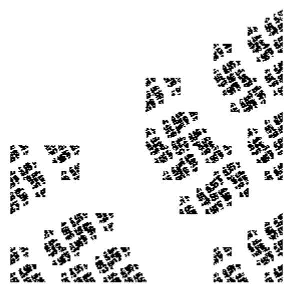
b.



c.

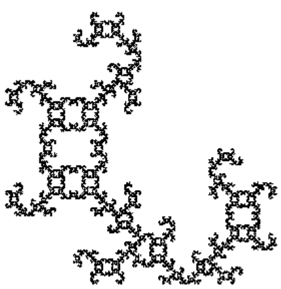


d.

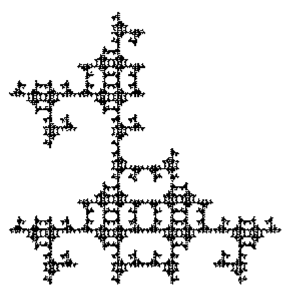


3. For each of the following fractals, draw the first iteration of an IFS that produces it, using the deterministic method starting with Mr. Face (i.e. play *Guess My IFS* with the following fractals).

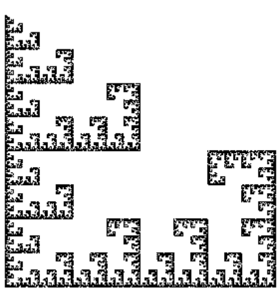
a.



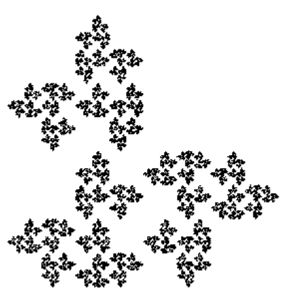
b.

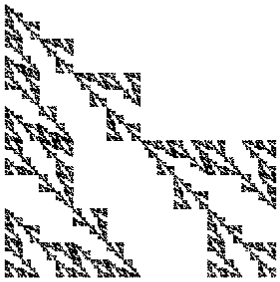


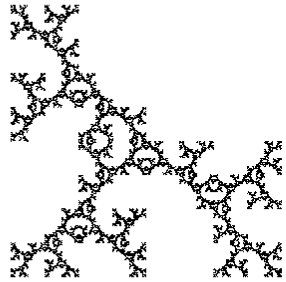
c.



d.



e. 

f. 

4. Determine the IFS numerical codes (i.e. the table of numbers like those in problem number 1) for the IFS which produced the fractal in:

a. part (a) of the previous problem.

b. part (b) of the previous problem.

Maple Makes Fractals

1. Go to the Math Lab, STT161. Run the program Maple 6 from the Start Menu. Press CTRL-T (to switch to text) and type your name and assignment number on the top of the Maple worksheet. [Note: your name and assignment # must be typed in the worksheet, not handwritten on it.] When you are done typing your name press CTRL-J to get a new prompt and then type:

```
with(chaos):
```

and press Enter. This loads my program for making fractals. To plot the attractor of HeeBGB(Up,Lt,Rt,none) you can use the following command:

```
DrawIFS(HeeBGB(Up,Lt,Rt,none),20000);
```

The number 20000 indicates that 20000 points should be used for the plot. Using more points will make the picture darker, but will take a lot longer. Don't use more than about 30000-40000 points or it may never finish. Now modify the above command to draw the fractals shown in problem #3a-f, in the IFS section above. [Hint: they are all HeeBGB fractals.] When you are finished, print your worksheet and hand in your printout as part of this assignment.

2. Make up any two HeeBGB fractals of your own and plot them using the same procedure you used in the previous problem. Note: Since you are making them up, no two students should have the same exact choice! I will not give credit for duplicates.

3. We can also plot the images of MrFace (and other seeds) using the Deterministic Method with an IFS. For example, to draw the 1st iteration of HeeBGB(Up,Lt,Rt,none) starting with MrFace we use the command:

```
DrawDetIFS(MrFace,HeeBGB(Up,Lt,Rt,none),1);
```

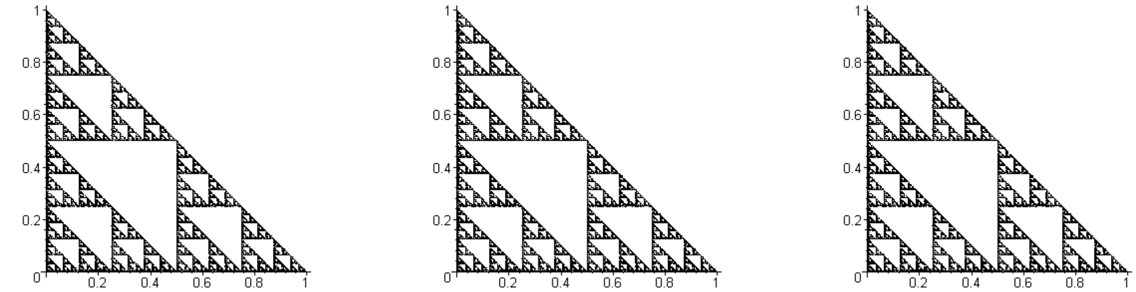
To draw the second iteration we would change the 1 to a 2, etc. Modify the above command to draw the first 4 iterations for IFS's that produce the fractals shown in problem #3a-b, in the IFS section above. When you are finished, print your worksheet and hand in your printout as part of this assignment.

4. Imitate the method of the previous problem to draw the first four or five iterations starting with MrFace, for the HeeBGB fractals you made up in problem #2 above.

Addresses

1. Consider the Sierpinski triangle produced by the following IFS:

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
T_1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0
T_2	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$

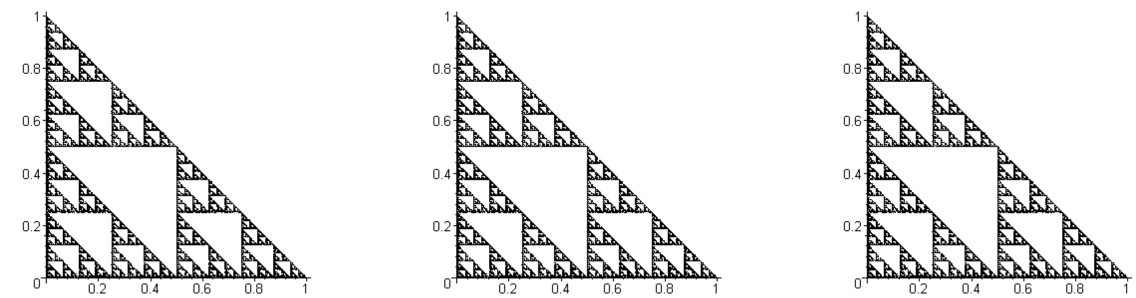


For each of the following, color and label all the points on the attractor above whose address begins with the digits:

a. 1
b. 02
c. 20
d. 201
e. 102
f. 0221
g. 2102

2. Repeat the previous problem, except using this IFS to generate the Sierpinski triangle:

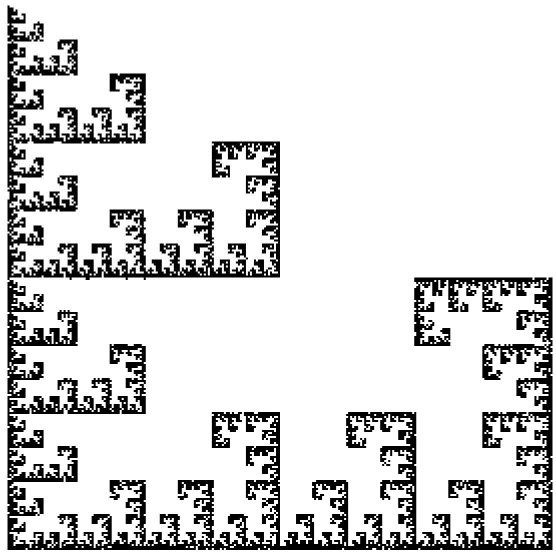
	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$
T_1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
T_2	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0



3. The following IFS

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
T_1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$
T_2	$\frac{1}{2}$	$\frac{1}{2}$	90	90	1	0

produces the following fractal



On the above picture, color and label all the points whose address begins with:

- **a.** 1
- **b.** 20
- **c.** 021
- **d.** 0122
- **e.** Label the point whose address is $02\overline{1}$.
- **f.** Give a different address for the same point in part e.
- **g.** Label the point whose address is $\overline{2}$.

4. Play the Chaos Game Game at http://math.bu.edu/DYSYS/applets/chaos-game.html (you can access this from one of the computer labs on campus if you don't have your own access to the internet). Play game number 1 (the Sierpinski Triangle, and play it on HARD level. Draw or print the results and explain how you beat it in the Best Possible Score using IFS addresses.

5. Consider the IFS

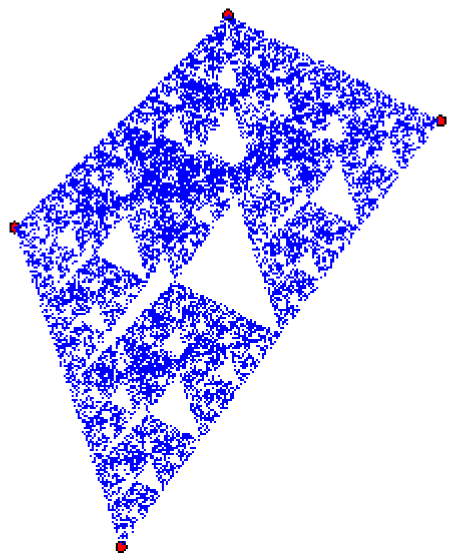
	r	s	θ	ϕ	e	f
T_0	$\frac{1}{3}$	0	0	0	0	0
T_2	$\frac{1}{3}$	0	0	0	$\frac{2}{3}$	0

which produces the Cantor set:

- **a.** Circle the points whose address begins with 022.
- **b.** Label the point whose address is $2\overline{0}$.

Random Iteration Method

1. Choose any four points, *A, B, C, D,* in the plane so that no three of the points lie on a straight line. These are your GOAL points. Plot these four points.
   a. Now choose any starting point, and plot it, and make this your initial input to the following process. Choose one of the four GOAL points at random (for example, if you flip two coins, then you can interpret HH=A, HT=B, TH=C, and TT=D where H means "Heads" and T means "Tails") and plot the point that is half way between your input put and the GOAL point you selected. Draw the first twenty iterates of this process (on a single plot), and number the points you plotted in the order you plot them: 0,1,2,3,4,... etc. [Note: if the four goal points are chosen to be the corners of a square, then this is the Four Corner Chaos Game.]
   b. I asked my computer to solve part (a) above, except that I let it plot the first 20,000 iterations instead of just the first 15 iterations. Here is the picture I got:



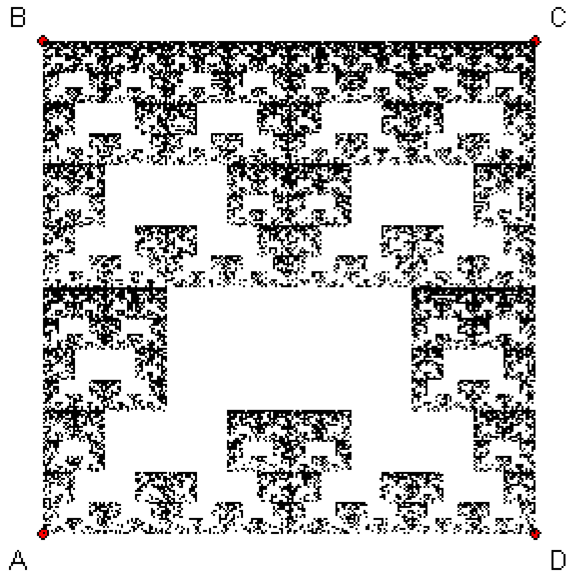
   Explain why there are shapes that resemble parts of Sierpinski triangles in the above picture.

Testing Sequences for Randomness

1. A genetic researcher has a strand of DNA which contains the following sequence:

   ACTTGCTAGCTGGGAGTCA.

   Use this sequence to play the Four Corner Chaos Game (see #1a above) starting with the point in the center of the square as your initial point and using the letters in the order they appear from left to right. Number the points you plot in the order you plot them.
2. A radio astronomer has a 10,000 character sequence of the letters A,B,C,D which he obtained from signals he received from outer space. The sequence appears to be random, but he decides to test it using the chaos game. He plots four points labeled A, B, C, and D at the four corners of a square and starting at the point A, he looks at the first letter in his sequence and moves half way towards the corner with that label. He then reads the next letter in the sequence and moves half way to the next corner, and so on in the usual manner for playing the chaos game. If the sequence was truly random, playing this game should fill in the square more or less uniformly. However, after plotting the 10,000 points from his sequence he obtained the following picture:

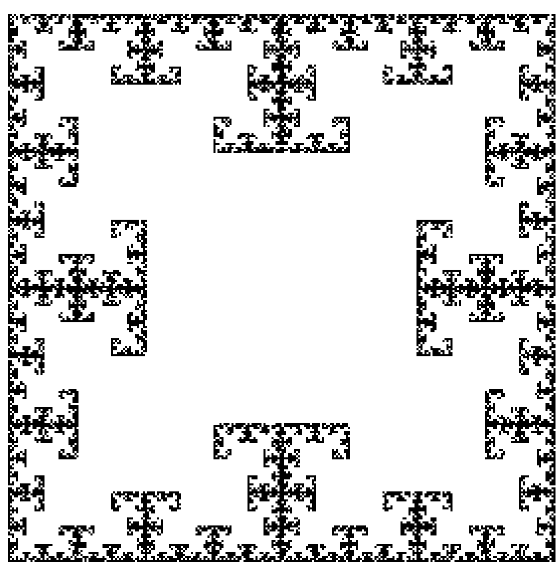


What does this picture indicate about the sequence of letters? In particular, what subsequences never appear in this sequence?

3. Maple can be used to plot restricted IFS's as well as ordinary IFS's. For example, to plot the restricted IFS that avoids the address subsequences 12, 21, 30, and 03, we can do the following:

```
with(chaos):
RestrictedIFS(HeeBGB(Up,Up,Up,Up),30000,{[1,2],[2,1],[3,0],[0,3]});
```

which produces the fractal:



- **a.** Follow the instructions in Problem #1 of the **Maple Makes Fractals** section above to put your name, course, and assignment # at the top of your Maple worksheet.
- **b.** Imitate the commands above to produce the attractor of the restricted IFS that avoids the address subsequences 00,11,22, and 33.
- **c.** Plot the restricted IFS that avoids the address subsequences 122, 211, 30 and 03.
- **d.** Plot the restricted IFS that avoids only the address subsequence 3. Explain the results.
- **e.** Make up a restricted IFS of your own and plot its attractor. Try to find something that looks

interesting.

Fractal Data Analysis

1. We can use the random iteration method to analyze numerical data to quickly see if it is truly random or if it has some structure to it.
   a. Flip two different coins (e.g. a nickel and a penny) to get a random number from 0 to 3 according to the following table:

Nickel	Penny	Value
H	H	0
H	T	1
T	H	2
T	T	3

For example if the nickel comes up tails and the penny comes up heads, you would record the value 2 for that flip. Do this at least fifty (50) times and record all your values (the more data you collect the better your results will be, so feel free to use more than 50 points if you can endure all the flipping without "flipping out"). Use the same pair of coins for all of your flips, do not switch coins in the middle of collecting data. Then use Maple to analyze your data as follows:

i. Put your name and assignment # on the top of the Maple sheet as usual.

ii. Enter your data using these commands:

```
with(chaos):
data:=[2,1,3,0,...(type your actual data here)...3,3,0,1,2]:
data:=map(x->x/4+0.125,data):
```

iii. Then analyze your data with the command:

```
ChaosGameTest(data,symbol=CIRCLE);
```

This will play the four corner chaos game with your data and plot the result.

iv. Do your coins appear to be random? If not, what can you say about the probabilities of flipping the four possible outcomes? Are 50 points enough to draw a good conclusion? If not, how many data points do you think you would need to get good results?

Logarithms

Note: You should do the problems in this section by hand (you will not be allowed to use calculators on the exams) but you can check your answers on a calculator.

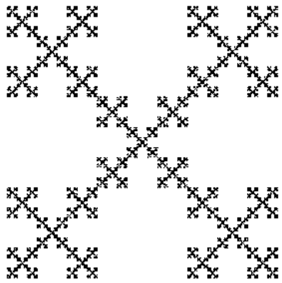
1. Evaluate the following expressions.
   a. 2^3
   b. 4^3
   c. 243^0
   d. 2^{-3}
   e. $\left(\frac{1}{3}\right)^4$
   f. $\left(\frac{1}{3}\right)^{-3}$
   g. $\left(\frac{3}{5}\right)^{-2}$
   h. 5^{-1}
   i. $\left(\frac{1}{3}\right)^{-1}$
   j. $\left(\frac{2}{7}\right)^{-1}$

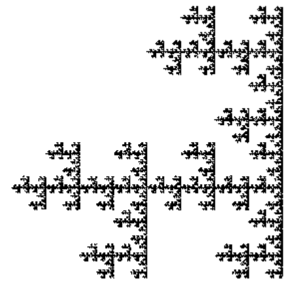
k. $(0.1)^2$
l. $(0.02)^3$
m. $(0.1)^{-3}$
n. $(0.5)^{-5}$

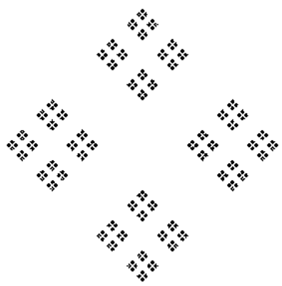
2. Evaluate the following expressions.
   a. $\log_3(9)$
   b. $\log_2(16)$
   c. $\log_3(81)$
   d. $\log_5(1)$
   e. $\log_3(3^{-2})$
   f. $\log_2(0.5)$
   g. $\log_6\left(\frac{1}{6}\right)$
   h. $\log_2\left(\frac{1}{32}\right)$
   i. $\log_9(1)$
   j. $\log_{10}\left(\frac{1}{100}\right)$
   k. $\log\left(\frac{1}{100}\right)$
   l. $\log(10)$
   m. $\log(0.01)$
   n. $\log(10000)$
3. Suppose $\log(a) = 0.3$ and $\log(b) = 0.6$. Use the properties of logarithms to determine:
   a. $\log(ab)$
   b. $\log(\frac{b}{a})$
   c. $\log\left(\frac{a}{b}\right)$
   d. $\log(a^3)$
   e. $\log(b^5)$
   f. $\log(10^{\log(a)})$

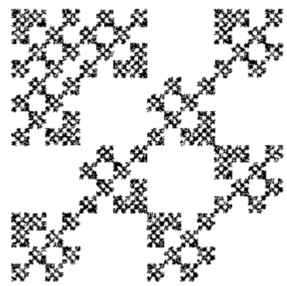
Similarity Dimension

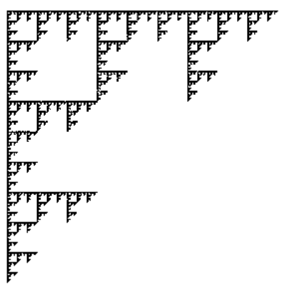
1. What is the topological dimension that best describes the following objects:
   a. a brick
   b. a shoelace
   c. a grain of sand
   d. a blanket
   e. power lines
   f. a window pane
   g. Mt. Rushmore
   h. an egg
   i. the shell of an egg
   j. dust
2. Compute the similarity dimension of the following fractals. You may want to use a ruler.

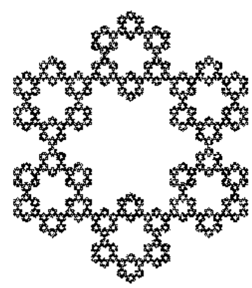
a. 

b. 

c. 

d. 

e. 

f.. 

3. Compute the similarity dimension of the fractals generated by the following IFS codes.

a.

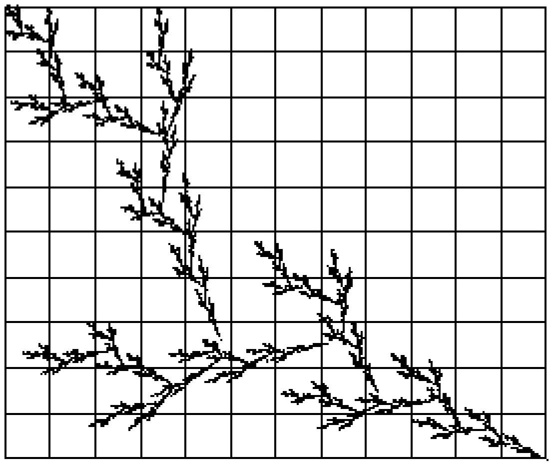
	r	s	θ	ϕ	e	f
T_0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
T_1	$\frac{1}{2}$	$\frac{1}{2}$	–90	–90	$\frac{1}{2}$	$\frac{1}{2}$
T_2	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$

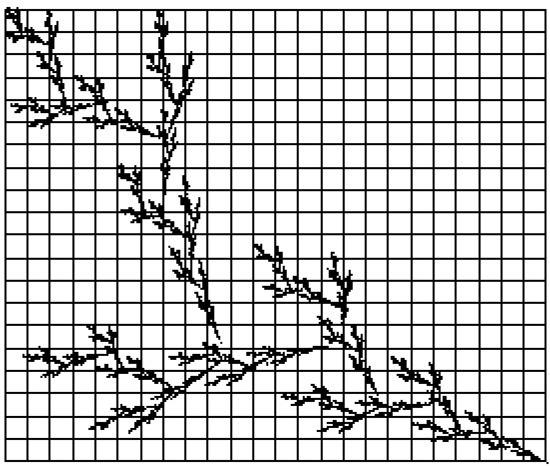
b.

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0
T_2	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0

Grid Dimension

1. Compute the grid fractal dimension of the fractal seaweed show in the two figures below. The distance between the grid lines in the first figure is 2, while the distance between the grid lines in the second figure is 1. Explain your work, don't just give a numerical answer.





Chaos

1. State and briefly explain the three properties of a chaotic dynamical system.

Note: in the next two questions you can use Maple to compute the orbits if you wish. For example, if you wish to have Maple compute the first 20 iterations of the L-orbit of 0.7001 in #2a you can use the commands:

```
with(chaos):
Orbitf( x->4*x*(1-x) , 0.7001 , 20);
```

So the `Orbitf` command takes three inputs, the first is the function, the second is the seed and the third is the number of iterations to compute. Notice that to specify the function $f(x)$ in Maple, you write it as `x->f(x)` meaning "x is sent to $f(x)$" where $f(x)$ is the expression that defines the function f. Thus $f(x) = x^2 - 2$ would be written `x->x^2-2` and $L(x) = 0.3x(1-x)$ would be written `x->0.3*x*(1-x)` in Maple.

Reminder: If you use Maple your printouts MUST be labeled with your name and assignment number as prescribed for Maple assignments above and all printouts and written work MUST be stapled if you want credit!

2. One example of a chaotic map is the logistic map $L(x) = 4x(1-x)$ (on the set of numbers between 0 and 1).

a. Since L is chaotic, it has sensitive dependence on initial conditions. Illustrate this by computing the L-orbits of the numbers 0.7 and 0.7001. How many iterations does it take before the corresponding terms in the orbits differ by at least 0.5?

b. A chaotic map is supposed to have dense periodic points, so there should be LOTS (infinitely many!) periodic points for L. Of course finding them is another question entirely! Here are some candidates I found. Notice that they are distributed throughout the entire interval of numbers between zero and one. For each of our candidates below, determine its (apparent) period. [Note: Do not round the digits or else you will get bitten by sensitive dependence on initial conditions. Use a calculator or Maple. Note that no matter how many digits of accuracy you use, you might not come back exactly to the starting number because of roundoff error. Thus, many of these points are not periodic themselves, but rather are very close to a periodic point.]

i. 0.0
ii. 0.0337638853
iii. 0.0432272712
iv. 0.1169777784
v. 0.1304955414
vi. 0.1654346968
vii. 0.1882550991
viii. 0.2771308221
ix. 0.3454915028
x. 0.4131759112
xi. 0.4538658203
xii. 0.5522642316
xiii. 0.6112604670
xiv. 0.6368314950
xv. 0.7500000000
xvi. 0.8013173182
xvii. 0.9045084972
xviii. 0.9251085679
xix. 0.9504844340
xx. 0.9698463104
xxi. 0.9890738004
xxii. 0.9914865498

c. Since L is chaotic it must have a transitive orbit. choose any number decimal between 0 and 1 to two decimal places of accuracy (e.g. 0.82, 0.31, etc). Choose whatever number you like. Then determine the number of iterations of the orbit of 0.42 needed before you number appears in the orbit to within two decimal places of accuracy.

3. Let $L(x) = cx(1-x)$ be the logistic map with growth constant c. Use a calculator to compute the first twenty iterations of the orbit of 0.3 for the logistic map for each of the following values of c. Do the orbits appear to be stable or unstable? If they appear to be stable, what are they approaching?

a. $c = 1$

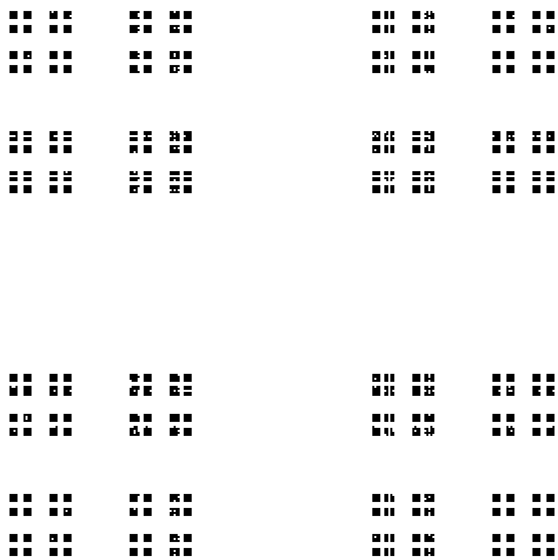
b. $c = 3.1$

c. $c = 3.9$

4. Consider the IFS,

	r	s	θ	ϕ	e	f
T_0	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0
T_1	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{2}{3}$	0
T_2	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{2}{3}$	$\frac{2}{3}$
T_3	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	$\frac{2}{3}$

. The fractal produced by this IFS is:



a. Compute the addresses of the points in the orbit of the shift map starting with the point whose address is $032210\overline{3}$

b. Draw the orbit in part (a) on the fractal image above (using arrows to connect the points as I did in class).

c. Repeat part (a) for the point whose address is $21330\overline{12}$.

d. Repeat part (b) for the point whose address is $21330\overline{12}$.

Complex Numbers

1. Use complex addition or subtraction to write the following expressions as a complex number in standard form.

a. $(1 + 2i) + (3 + 5i)$

b. $(4 + 2i) + (3 - 5i)$

c. $(4 + 2i) - (3 - 5i)$

d. $(4 + i) + (-3 + 3i)$

e. $(2 + i) - 3$

f. $2i + (4 - i)$

g. $(5) + (3i)$

h. $(2 - i) + (-2 + i)$

i. $\left(\frac{1}{2}+2i\right)-\left(\frac{3}{2}+\frac{2}{5}i\right)$
j. $\left(-3-\frac{1}{3}i\right)+\left(3-\frac{1}{3}i\right)$
k. $(0.23+3.41i)+(0.26-2.93i)$
l. $(-1.23+0.51i)-(-0.62-1.37i)$
m. $(0.39845+0.97133i)-0.22987i$

2. Use complex multiplication to write the following expressions as a complex number in standard form.
   a. $(1+2i)(3+5i)$
   b. $(4+2i)(3-5i)$
   c. $(4+2i)(3-5i)$
   d. $(4+i)(-3+3i)$
   e. $3(2+i)$
   f. $2i(4-i)$
   g. $(5)(3i)$
   h. $(2-i)(-2+i)$
   i. $\left(\frac{1}{2}+2i\right)\left(\frac{3}{2}+\frac{2}{5}i\right)$
   j. $\left(-3-\frac{1}{3}i\right)\left(3-\frac{1}{3}i\right)$
   k. $(0.23+3.41i)(0.26-2.93i)$
   l. $(-1.23+0.51i)(-0.62-1.37i)$
   m. $-(0.39845+0.97133i)0.22987i$
3. Use complex arithmetic to write the following expressions as a complex number in standard form.
   a. $(1+2i)^2$
   b. $(4+2i)^2+(3-5i)$
   c. $-\left(-3-\frac{1}{3}i\right)^2+\left(3-\frac{1}{3}i\right)$
   d. $(0.39845+0.97133i)+(0.22987i)^2$
4. Write the following expressions as a complex number in standard form.
   a. i^2
   b. i^3
   c. i^4
   d. i^5
   e. i^6
   f. i^7
   g. i^8
5. Compute the following.
   a. $|(1+2i)|$
   b. $|(3-5i)|$
   c. $|(1+2i)(3-5i)|$
   d. $|2i|$
   e. $|-5|$
   f. $\left|\left(\frac{3}{2}+\frac{2}{5}i\right)\right|$
   g. $|(0.23+3.41i)|$
   h. $|(0.39845+0.97133i)|$
6. For each seed, $z_0,$ and function given below, compute the orbit. (If the orbit is not periodic or eventually periodic compute the first six iterations.) In each case state whether the orbit is cyclic,

eventually cyclic, or neither? If it is neither, is the orbit attracted to a cycle?

a. $f(z) = 2z,\ z_0 = i$
b. $g(z) = i + z,\ z_0 = 1 + i$
c. $h(z) = -z,\ z_0 = -2 + 3i$
d. $L(z) = iz,\ z_0 = 0$
e. $L(z) = iz,\ z_0 = 1$
f. $S(z) = z^2 - 2,\ z_0 = -0.4$
g. $r(z) = z^2,\ z_0 = -i$
h. $g(z) = z^2 - i,\ z_0 = 1$

Mandelbrot and Julia Sets

1. Plot each of the following complex numbers on the complex plane.
   a. 0
   b. 2
   c. -3
   d. i
   e. $-3i$
   f. $1 + 2i$
   g. $-3 - 2i$
   h. $-4 - i$
   i. $\frac{1}{2} + 3i$
   j. $0.25 + 1.75i$
2. Plot the orbits you computed in problem #6 of the previous section on the complex plane (plot the points connected by arrows).
3. State the definition of each of the following fractals.
   a. the Mandelbrot set, M
   b. the filled in Julia set, K_c
   c. the Julia set, J_c
4. For each of the following complex numbers c, determine if it is in the Mandelbrot set M by computing the Q_c-orbit of 0 until you get a value whose absolute value is greater than 2, or you determine that the orbit is bounded. Do these by hand, don't use Maple.
   a. 0
   b. -1.754877666
   c. 5
   d. -1
   e. i
   f. $1 + i$
   g. -2
   h. $-1 + i$
   i. 0.5
   j. $-3i$
5. For each of the complex numbers c in problem #3, determine if the Julia set J_c is connected.
6. Plot all of the complex numbers in problem #3 which are in the Mandelbrot set on a single plane.
7. For each of the following pairs of complex numbers, z and c, determine if z is in the filled in Julia set K_c. Do these by hand, don't use Maple.

a. $z = 0,\ c = i$

b. $z = -1 - i,\ c = i$

c. $z = 0,\ c = 0$

d. $z = 1,\ c = 0$

e. $z = 2,\ c = 0$

f. $z = 0,\ c = 1 + I$

g. $z = 1 - I,\ c = 1 + I$

h. $z = I,\ c = 1 + I$