Practice Problems (answers)

Math 104 - Mathematics for Elementary Teachers
University of Scranton
Dr. Monks

1. 1

2. $^{-}109$

3. $16, 8, 4, 2, 1, \frac{1}{2}, \ldots$

4. **a.** T
b. T
c. T
d. T
e. F

5. 29, 25

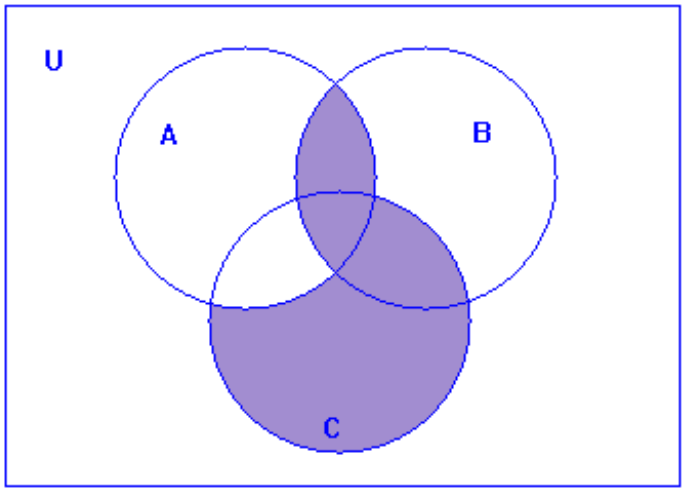
6. 21, 34

7. 180300

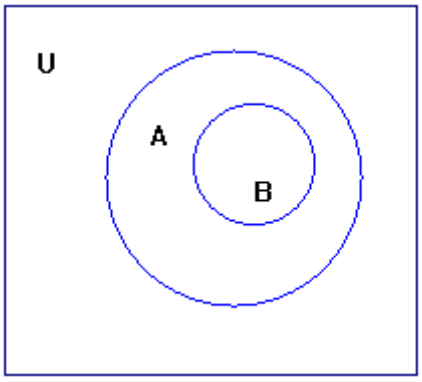
8. **a.** T
b. F
c. T
d. T
e. T
f. F
g. T
h. T

9. **a.** $\{0, 1, 2, 3, 4, 6, 8\}$
b. $\{0, 2, 4\}$
c. $\{(0,0), (0,2), (0,4), (0,6), (0,8),$
$(1,0), (1,2), (1,4), (1,6), (1,8),$
$(2,0), (2,2), (2,4), (2,6), (2,8),$
$(3,0), (3,2), (3,4), (3,6), (3,8),$
$(4,0), (4,2), (4,4), (4,6), (4,8)\}$
d. $\{1, 3\}$
e. $\{6, 8\}$
f. $\{5, 6, 7, 8, 9\}$
g. U
h. $\{\ \}$
i. $\{\ \}$
j. $\{1, 3, 6, 8\}$

10.



11.

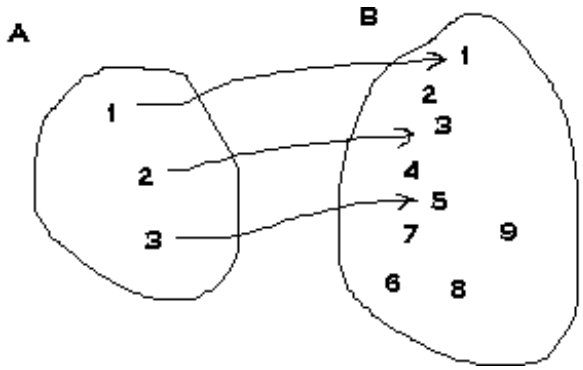


12. 90

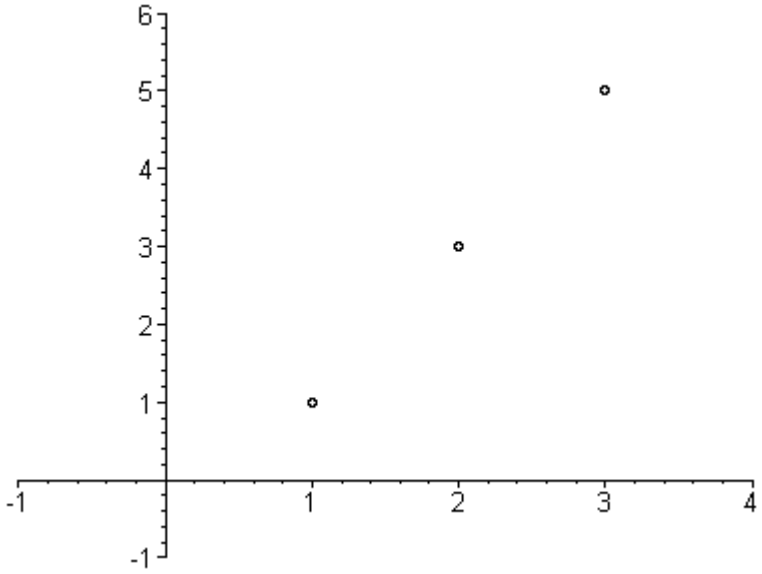
13. **a.**

x	$f(x)$
1	1
2	3
3	5
4	7

b.



c.



14. a, b, e, g, h, and i are statements, the rest are not.

15. **a.** If my grade is an A then my score is 100%.

b. If my grade is not an A then my score is not 100%.

c. If my score is not 100% then my grade is not an A.

d. My score is not 100%.

16.

P	Q	not(P)	not(Q)	Q or not(P)	P implies not(Q)	(P implies not(Q)) and (Q or not(P))
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

17. **a.** (ask me in class)

b. DXLIX

c. $670_{(9)}$

d. $20211_{(4)}$

18. 1599

19. **a.** 90

b. 242

c. 174

20. (all in base four)

+	0	1	2	3
0	0	1	2	3
1	1	2	3	10
2	2	3	10	11
3	3	10	11	12

and

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

a. $10331032_{(4)}$

b. $202233_{(4)}$

21. **a.**

$$\begin{aligned}
234 \cdot 45 &= (200 + 30 + 4) \cdot 45 \\
&= 200 \cdot 45 + 30 \cdot 45 + 4 \cdot 45 \\
&= 200 \cdot (40 + 5) + 30 \cdot (40 + 5) + 4 \cdot (40 + 5) \\
&= 200 \cdot 40 + 200 \cdot 5 + 30 \cdot 40 + 30 \cdot 5 + 4 \cdot 40 + 4 \cdot 5 \\
&= 8000 + 1000 + 1200 + 150 + 160 + 20 \\
&= 10530
\end{aligned}$$

b.

$$\begin{array}{r}
2\ 3\ 4 \\
\times\ 4\ 5 \\
\hline
2\ 0 \\
1\ 5\ 0 \\
1\ 0\ 0\ 0 \\
1\ 6\ 0 \\
1\ 2\ 0\ 0 \\
8\ 0\ 0\ 0 \\
\hline
1\ 0\ 5\ 3\ 0
\end{array}$$

c. I can't type the lattice, so ask me to see this one if you don't know how to do it.

d. (the rows marked with an → are supposed to be crossed out:)

→	234	45
	117	90
→	58	180
	29	360
→	14	720
	7	1440
	3	2880
	1	5760
		10530

e. (with appropriate "carries" not shown:)

$$\begin{array}{r} 234 \\ \times\ 45 \\ \hline 1170 \\ 9360 \\ \hline 10530 \end{array}$$

22. The answer is $372 = 83 \cdot 4 + 40$ as shown by the following methods.

a.

$$\begin{array}{r} 372 \\ -\ 83 \\ \hline 289 \\ -\ 83 \\ \hline 206 \\ -\ 83 \\ \hline 123 \\ -\ 83 \\ \hline 40 \end{array}$$

b.

$$\begin{array}{r|l} & 4 \\ \hline 83 & 372 \\ & 332 \\ \hline & 40 \end{array}$$

23. $3 + 4 = 7$ and 4 is a positive integer, so $3 < 7$ by the definition of $<$.

24. No. 2 and 4 are natural numbers, but $2 - 4$ is not a natural number, for example.

25. 26

26. **a.** $\gcd(27,66) = \gcd(12,27) = \gcd(3,12) = \gcd(0,3) = 3$

b. The divisors of 27 are 1,3,9,27. The divisors of 66 are 1,2,3,6,11,66. Thus $\gcd(27,66) = 3$.

c. $27 = 3^3$ and $66 = 2 \cdot 3 \cdot 11$ so $\gcd(27,66) = 3$.

27. **a.** $\gcd(27,66) = 3$ from above so $\text{lcm}(27,66) = \frac{27 \cdot 66}{\gcd(27,66)} = 594$.

b. The multiples of of 27 are

$$27, 54, 81, 108, 135, 162, 189, 216, 243, 270, 297,$$
$$324, 351, 378, 405, 432, 459, 486, 513, 540, 567, 594$$

while the multiples of 66 are

$$66, 132, 198, 264, 330, 396, 462, 528, 594$$

so $\text{lcm}(27,66) = 594$.

c. $27 = 3^3$ and $66 = 2 \cdot 3 \cdot 11$ so $\text{lcm}(27,66) = 2 \cdot 3^3 \cdot 11 = 594$.

28. It is not divisible by any of the numbers.

29. $2^{12}3^4 5^2 7^2$

30. **a.** identity of addition

b. distributive law

c. associative law of addition

d. commutative law of addition

e. reflexive law of equality

31. **a.** $^-12$

b. $^-3$

c. 12

d. $^-7$

e. 3

f. 7

g. $^-3$

32. **a.** T

b. F

c. F

d. T

e. F

f. T

33. 1,2,4,7,14,28

34. 37

35. 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59

36. $\frac{17}{23}$

37. $\frac{15}{23}$

38. $\frac{26}{15}$

39. $\frac{5}{7}$ inches

40. Yes, he is correct because

$$\left(6\frac{2}{3}\right) \times 3 = \left(6 + \frac{2}{3}\right) \times 3 \qquad \text{(by the definition of mixed number)}$$
$$= 6 \times 3 + \frac{2}{3} \times 3 \qquad \text{(by the distributive law)}$$
$$= 18 + 2$$
$$= 20$$

41. **a.** associativity of $+$
b. commutativity of $+$
c. commutativity of $\times$
d. multiplicative inverse
e. associativity of $\times$
f. identity of $\times$
g. identity of $+$
h. distributive law

42. **a.** $\frac{86}{39}$
b. $\frac{214}{105}$
c. $\frac{1829}{909}$
d. $\frac{13}{4}$

43. **a.** $1.113_{(5)}$
b. $1.2212_{(3)}$
c. $1.85_{(9)}$
d. $.\overline{052631578947368421}$

44. 0.1231

45. $00 : 25 : 37.9$

46. **a.** $\$2.21$
b. 11.43 lbs (to the nearest hundreth)

47. **a.** 11.04
b. $266.\overline{6}$
c. 150

48. **a.** 31547000
b. $^{-}20142.4543$
c. 0.00031547
d. $^{-}0.000000201424543$
e. 2.5

49. **a.** 2.4354×10^{-5}
b. 2.34565123×10^{8}
c. 2.3452356×10^{3}
d. $^{-}1.53 \times 10^{-4}$
e. 1×10^{-8}
f. $3.\overline{3} \times 10^{-1}$

50. **a.** T
b. F

c. F
d. T
e. T
f. F
g. T
h. T
i. F

51. a. $\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$
b. $\{\mathbb{I}, \mathbb{R}\}$
c. $\{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$
d. $\{\ \ \}$
e. $\{\mathbb{Q}, \mathbb{R}\}$
f. $\{\mathbb{Q}, \mathbb{R}\}$
g. $\{\mathbb{Q}, \mathbb{R}\}$
h. $\{\mathbb{Q}, \mathbb{R}\}$
i. $\{\mathbb{I}, \mathbb{R}\}$
j. $\{\mathbb{I}, \mathbb{R}\}$
k. $\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$
l. $\{\mathbb{Q}, \mathbb{R}\}$
m. $\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$
n. $\{\mathbb{I}, \mathbb{R}\}$
o. $\{\mathbb{I}, \mathbb{R}\}$
p. $\{\mathbb{Q}, \mathbb{R}\}$

52. a. $\mathbb{N}$
b. $\mathbb{Q}$
c. $\emptyset$
d. $\mathbb{R}$
e. $\mathbb{R}$
f. $\mathbb{Q}$
g. $\mathbb{Z}$
h. $\mathbb{R}$
i. $\mathbb{N}$
j. $\emptyset$
k. $\mathbb{I}$
l. $\mathbb{Q}$

53. a,b,e,f,g,j,k

54. a. commutativity of $+$
b. commutativity of $\times$
c. associativity of $\times$
d. identity of $\times$
e. distributive law

f. identity of +

g. associative of +

55. **a.** $2\sqrt{14}$

b. 6

c. $\frac{3}{7}\sqrt{7}$

d. $\frac{1}{2}$

56. $s = 7p$

57. **a.** $m + c$

b. $1.35c$

c. $1.20m + 1.35c$

58. **a.**

$$3(x-1) = 4 + 2x$$

$$3x - 3 = 4 + 2x \qquad \text{Simplify}$$

$$3x - 3 + (^-2x + 3) = 4 + 2x + (^-2x + 3) \qquad \text{Add equals}$$

$$x = 7 \qquad \text{Simplify}$$

Thus the solution set is $\{7\}$.

b.

$$3(x-1) < 4 + 2x$$

$$3x - 3 < 4 + 2x \qquad \text{Simplify}$$

$$3x - 3 + (^-2x + 3) < 4 + 2x + (^-2x + 3) \qquad \text{Add equals}$$

$$x < 7 \qquad \text{Simplify}$$

Thus the solutions set is the $\{x : x < 7\}$.

c.

$$\tfrac{3x+1}{x} = 7$$

Assume $x \neq 0$

$$3x + 1 = 7x \qquad \text{Multiply equals}$$

$$3x + 1 + (^-7x - 1) = 7x + (^-7x - 1) \qquad \text{Add equals}$$

$$^-4x = {}^-1 \qquad \text{Simplify}$$

$$(^-4x)\left(-\tfrac{1}{4}\right) = (^-1)\left(-\tfrac{1}{4}\right) \qquad \text{Multiply equals}$$

$$x = \tfrac{1}{4} \qquad \text{Simplify}$$

If $x = 0$ then $\frac{3x+1}{x} = 7$ is not defined, so the solution set is $\left\{\frac{1}{4}\right\}$.

d.

$$\tfrac{3}{x} = \tfrac{4}{x}$$

Assume $x \neq 0$

$$\tfrac{3}{x}x = \tfrac{4}{x}x \qquad \text{Multiply equals}$$

$$3 = 4 \qquad \text{Simplify}$$

If $x = 0$ then $\frac{3}{x} = \frac{4}{x}$ is not defined, thus since the equation $3 = 4$ has no solutions, the solution set is the empty set, $\{\ \}$.

59. **a.** $\{0\}$

b. $\{\ \ \}$

c. $\mathbb{R}$

d. $\{0\}$

e. $\mathbb{R}$

f. $\{0\}$

g. $\mathbb{R}$

h. $\{\ \ \}$

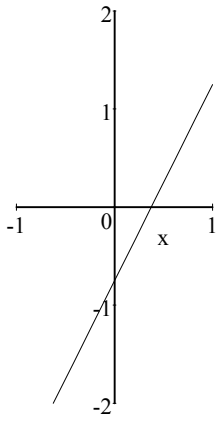
60. a. No. When $n = 1$, $2n = 2$ and $n + 2 = 3$ so, $2n$ is less than $n + 2$ when n is 1.

b. 761

61. a. $y = -\frac{9}{7}x + \frac{24}{7}$

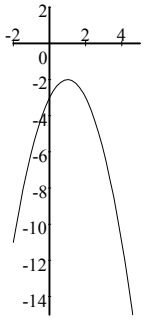
b. $y = -3x$

c. $y = 2x - \frac{3}{4}$

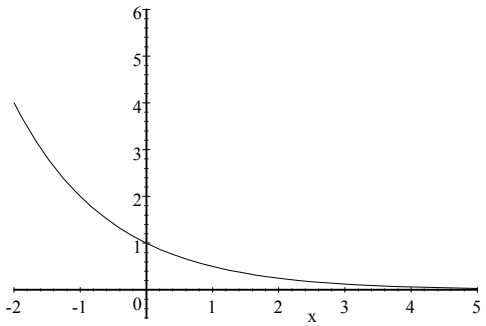


d. The slope= $\frac{1}{3}$. The y-intercept is 5.

62. a. $y = -x^2 + 2x - 3$



b. $y = 2^{-x}$



63. **a.** \$5362.54
b. \$7095.33
c. \$10068.76
d. \$20275.99
e. \$165577.25

64. \$594520.54

65. **a.** $\frac{1}{2}$
b. $\frac{3}{4}$
c. $\frac{1}{4}$
d. $\frac{5}{18}$
e. $\frac{1}{6}$
f. $\frac{5}{6}$
g. $\frac{1}{6}$

66. \$0.50

67. **a.** $\frac{4}{15}$
b. $\frac{2}{5}$

68. $\frac{26}{35}$

69. **a.** $\frac{1}{13}$
b. $\frac{1}{4}$
c. $\frac{1}{2}$
d. $\frac{11}{26}$
e. $\frac{3}{52}$

70. **a.** 1 to 999999
b. 1 to 24
c. 9 to 1

71. **a.** $\frac{4}{7}$
b. $\frac{1}{5}$

72. **a.** (done in class)
b. $\frac{3}{4}$

c. $\frac{7}{12}$

d. $\frac{1}{4}$

73. $\frac{2465}{6561}$

74. 7

75. See lecture notes.

76. **a.** T

b. F

c. T

d. T

e. T

f. T

g. T

77. **a.** $\overline{DB}$, $\overline{BE}$, $\overline{EC}$, $\overline{CA}$, $\overline{AB}$, $\overline{DE}$, $\overline{BC}$, $\overline{DC}$

b. $\angle DBA$ is obtuse, $\angle ABC$ is acute

c. $\overleftrightarrow{DC}$

d. $\triangle ABC$

78. 1620

79. $\left(\frac{900}{7}\right)°$ or approximately $128.6°$ each.

80. 8

81. A square is convex. For a nonconvex one is shown in section 21 of the lecture notes.

82. **Alternate interior angle theorem**: If two lines are cut by a transversal, the pairs of alternate interior angles formed are congruent if and only if the lines are parallel.

Vertical angle theorem: Vertical angles are congruent.

83. **a.** 142°

b. 38°

c. 26°

d. 43°

e. 107°

f. 73°

g. 116°

h. 38°

i. 142°

84. A figure eight.

85. The angles of a decagon have measure $\frac{180(10-2)}{10} = 144°$. If there was regular tesselation of the plane using decagons, there would have to be a whole number k of these angles meeting at a single vertex. Since the angle sum at the vertex would be 360°. So $360 = 144k$ for some whole number k. But this cannot happen because 144 is not a divisor of 360.

86. $\frac{1}{2}rh + \frac{1}{2}(r+s)k$

87. **a.** $160 + \frac{25}{2}\pi$

b. $42 + 5\pi$

88. Area= $\frac{1}{2}rs$, Perimeter= $r + s + \sqrt{r^2 + s^2}$

89. $\pi s^2 - \pi r^2$

90. 3.9π in^3 or approximately 12.3 in^3.

91. Volume$\cong 1.08 \times 10^{21}$ m^3, Area$\cong 5.1 \times 10^{14}$ m^2

92. $\frac{\sqrt{3}}{4} s^2 h$

93. 2940π ft^3 or approximately 9236.3 ft^3