Practice Problems

Math 104 - Mathematics for Elementary Teachers
University of Scranton
Dr. Monks

Instructions: You are not allowed to use calculators. Show all of your work.

1. Find the one millionth term in the periodic sequence
$$\overline{1,2,3}$$
(note: the first term is 1).
2. Find the 57^{th} term in the arithmetic sequence $3, 1, {}^{-}1, {}^{-}3, \ldots$.
3. Write the first six terms of the geometric sequence whose first term is 16 and whose common ratio is $\frac{1}{2}$.
4. True or False
   a. Every periodic sequence is an infinite sequence.
   b. A finite sequence can have more than ten terms.
   c. The constant sequence $3, 3, 3, 3, \ldots$ is both arithmetic and geometric.
   d. The sequences $1, \overline{2,3,1}$ and $\overline{1,2,3}$ represent the same sequence.
   e. If the common difference in an arithmetic sequence is equal to 1 then the sequence is a constant sequence.
5. Use the Method of Finite Differences to find the next two terms in the sequence
$$1, 4, 9, 15, 21, 26, 29, \ldots$$
6. Compute the next three terms of the Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, \ldots$.
7. Compute the sum of the integers from 1 to 600 inclusive, i.e. compute $1 + 2 + 3 + 4 + \cdots + 598 + 599 + 600$.
8. True or False
   a. $3 \in \{1,2,3,4,5\}$
   b. $3 \subseteq \{1,2,3,4,5\}$
   c. $\{3\} \notin \{1,2,3,4,5\}$
   d. $\{3\} \subseteq \{1,2,3,4,5\}$
   e. $\{3,2,5\} = \{5,3,2\}$
   f. $\{3,2,5\}$ is a proper subset of $\{3,2,5\}$
   g. $2 \in \{x : x \text{ is an even integer}\}$
   h. $2 \in \{x : x^2 = 4\}$
9. Let $U = \{0,1,2,3,4,5,6,7,8,9\}$ be the universal set and let $A = \{0,1,2,3,4\}$, $B = \{0,2,4,6,8\}$, and $C = \{\}$ be subsets of U. Compute the following sets (in finite set enumeration notation).
   a. $A \cup B$
   b. $A \cap B$
   c. $A \times B$
   d. $A - B$
   e. $B - A$
   f. A'

g. C'

h. $(A \cup C) \cap C$

i. $C - B$

j. $(A - B) \cup (B - A)$

10. Draw a Venn diagram to illustrate the set $(A \cap B) \cup (C - A)$.

11. Draw a Venn diagram to illustrate that a set B is a subset of another set A.

12. Out of 100 freshmen at Lance College, 60 are taking English, 50 are taking history, 30 are taking mathematics, 30 are taking both English and history, 16 are taking both English and mathematics, 10 are taking both history and mathematics, and 6 are taking all three. How many of the freshmen are enrolled in English, history, or mathematics?

13. Consider the function $f : \{1,2,3,4\} \to \{1,2,3,4,5,6,7,8,9,10\}$ defined by the rule $f(x) = 2x - 1$.

a. represent this function as a table

b. represent this function by a Venn-like diagram

c. draw the graph of the function

14. Which of the following is a statement?

a. Joe is short.

b. I'm cold.

c. Oh, my!

d. Are you tired?

e. $3 = 7$

f. $6 \div 3$

g. $6 \mid 3$

h. $3 \in \{1,2\}$

i. $3 < {}^{-}12$

15. Consider the statement "If my score is 100% then my grade is an A".

a. Write the converse of the statement.

b. Write the contrapositive of the statement.

c. Write the inverse of the statement.

d. Use the Law of Contraposition to deduce a conclusion from the following two statements:

1. If my score is 100% then my grade is an A
2. My grade is not an A.

16. Make the truth table for $\big(\big(P \text{ implies not}(Q)\big) \text{ and } \big(Q \text{ or not}(P)\big)\big)$.

17. Express the number 549

a. in Mayan.

b. as a Roman numeral.

c. in base 9.

d. in base 4.

18. Express the Roman numeral MDXCIX in base ten.

19. Convert the following to base ten.

a. $1011010_{(2)}$

b. $1432_{(5)}$

c. $AE_{(16)}$

20. Make the addition and multiplication tables for base four.

a. Compute the sum of $1023213_{(4)} + 3301213_{(4)}$ (by working in base four, do not convert anything to base ten).

b. Compute the product of $321_{(4)} \times 213_{(4)}$ (by working in base four, do not convert anything to base ten).

21. Evaluate the product 234×45 by:

a. the horizontal method

b. the vertical method

c. the lattice method

d. the Russian peasant method

e. the traditional method

22. Find the integer quotient and remainder when 372 is divided by 83 and write your answer in the form indicated by the Division Theorem using:

a. the repeated subtraction method

b. the traditional method

23. Prove that $3 < 7$.

24. Is the set of natural numbers closed under subtraction? Explain your answer.

25. Compute $2 + 3 \cdot 2^3$.

26. Compute $\gcd(27, 66)$

a. by the Euclidean algorithm

b. by listing divisors

c. by prime factorization

27. Compute $\text{lcm}(27, 66)$

a. by the Euclidean algorithm

b. by listing multiples (warning... lots of work!)

c. by prime factorization

28. Use divisibility tests to determine if 123456789101112131415161718 19 is divisible by:

a. 2

b. 3

c. 4

d. 5

e. 6

f. 7

g. 8

h. 9

i. 10

j. 11

(Reminder: show your work for each of the tests, do not just say yes or no!).

29. Use a factor tree to find the prime factorization of 406425600.

30. For each of the following sentences, name the property of the integers which explains why the sentence true.

a. $(3+4)+0=(3+4)$

b. $(3\cdot 4+3\cdot 3)=3\cdot(4+3)$

c. $(3+4)+0=3+(4+0)$

d. $(3+4)+0=0+(3+4)$

e. $(3+4)\cdot(4+4)=(3+4)\cdot(4+4)$

31. For each of the following expressions, find the integer which is equal to the expression.

a. $(^-3)\cdot(^-2)\cdot(^-2)$

b. $(2+(^-5))$

c. $(^-3)\cdot(^-2)\cdot(2)$

d. $(^-2+(^-5))$

e. $(^-2-(^-5))$

f. $(2-(^-5))$

g. $(2-5)$

32. True or False.

a. For every integer n, if $6|n$ then $3|n$.

b. For every integer n, if $3|n$ then $6|n$.

c. For every pair of integers, a, b, if $6|(a\cdot b)$ then $6|a$ or $6|b$.

d. $2|0$.

e. For every integer n, it is the case that $n|n$.

f. For every pair of integers a, b, it is the case that $a|(a\cdot b)$.

33. List all of the positive divisors of 28.

34. What is the greatest prime number you need to consider as a possible factor when testing whether or not 1373 is a prime number?

35. Use the Sieve of Eratosthenes to determine the all of the prime numbers less than 60. Use the list below:

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	

36. Reduce $\frac{493}{667}$.

37. Which is larger, $\frac{15}{23}$ or $\frac{19}{30}$?

38. Compute $\frac{11}{6}+\frac{2}{4}-\frac{3}{5}$ and express you answer as a reduced fraction.

39. On Wednesday it rained $2\frac{1}{7}$ inches and on Thursday it rained only $\frac{1}{3}$ as much as it had on Wed. How many inches did it rain on Thursday?

40. Little Johnny mutiplied $6\frac{2}{3}$ by 3 as follows. First he computed 6 times 3 to get 18, then he multiplied $\frac{2}{3}$ times 3 to get 2, and finally he added the 18 to the 2 to get a final answer of 20. Is his method correct? If not, what is he doing wrong? If so, what laws and definitions of arithmetic can be used to justify this procedure?

41. State the property of rational numbers that justifies the following statements.

a. $\frac{1}{3}+\left(\frac{2}{5}+3\right)=\left(\frac{1}{3}+\frac{2}{5}\right)+3$

b. $\left(\frac{2}{5}+3\right)+\frac{1}{3}=\frac{1}{3}+\left(\frac{2}{5}+3\right)$

c. $\left(\frac{2}{5}\times 3\right)\times\frac{1}{3}=\frac{1}{3}\times\left(\frac{2}{5}\times 3\right)$

d. $3 \times \frac{1}{3} = 1$

e. $\frac{1}{3} \times \left(\frac{2}{5} \times 3\right) = \left(\frac{1}{3} \times \frac{2}{5}\right) \times 3$

f. $\left(\frac{2}{5} + 3\right) \times 1 = \left(\frac{2}{5} + 3\right)$

g. $\left(\frac{2}{5} + 3\right) + 0 = \left(\frac{2}{5} + 3\right)$

h. $1 \times \left(\frac{2}{5} + 3\right) = \left(1 \times \frac{2}{5} + 1 \times 3\right)$

42. Convert each of the following to a reduced fraction in base ten.

a. $2.0\overline{121}_{(3)}$

b. $2.01\overline{21}_{(6)}$

c. $2.\overline{0121}$

d. $11.00\overline{1}_{(2)}$

43. Convert the following.

a. Convert $\frac{158}{125}$ to its base 5 representation.

b. Convert $\frac{158}{81}$ to its base 3 representation.

c. Convert $\frac{158}{81}$ to its base 9 representation.

d. Convert $\frac{1}{19}$ to a repeating decimal. Make sure you place the overbar over the repeating part.

44. Find a decimal number that is between 0.123 and 0.1234.

45. Janet ran the Boston Marathon in 3:39:23.6 and Agatha ran it in 4:05:01.5. What was the difference between their times (in hours:min:sec)?

46. The price of chicken is proportional to its wieght.

a. If 2.4 pounds of chicken costs \$1.47, how much does 3.6 pounds of chicken cost to the nearest cent?

b. If 2.4 pounds of chicken costs \$1.47, how many pounds of chicken can you buy for \$7?

47. Answer the following. Do not round your answers.

a. What is 23% of 48?

b. 48 is what percentage of 18?

c. 48 is 32% of what number?

48. Express as a decimal number in base ten.

a. 3.1547×10^{7}

b. $^{-}2.01424543 \times 10^{4}$

c. 3.1547×10^{-4}

d. $^{-}2.01424543 \times 10^{-7}$

e. 2.5×10^{0}

49. Express in scientific notation.

a. 0.000024354

b. 234565132

c. 2345.2356

d. $^{-}0.000153$

e. $\frac{1}{10^{8}}$

f. $\frac{1}{3}$

50. Let $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{I}$, and $\mathbb{R}$ be the set of natural numbers, integers, rational numbers, irrational

numbers, and real numbers respectively, and let $\emptyset$ be the empty set. State whether each of the following is true or false.

a. $\mathbb{N} \subseteq \mathbb{Q}$

b. $\mathbb{I} \subseteq \mathbb{Q}$

c. $\mathbb{Q} \subseteq \mathbb{I}$

d. $\mathbb{I} \subseteq \mathbb{R}$

e. $\mathbb{Z} \subseteq \mathbb{Q}$

f. $\mathbb{Z} \subseteq \mathbb{N}$

g. $\mathbb{R} \subseteq \mathbb{R}$

h. $\emptyset \subseteq \mathbb{N}$

i. $\mathbb{N} \subseteq \emptyset$

51. For each of the following numbers list all of the sets in $\{\emptyset, \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{I}, \mathbb{R}\}$ that contain the number.

a. 2

b. $\sqrt{2}$

c. $^{-}2$

d. $\sqrt{^{-}2}$

e. $\frac{1}{2}$

f. $\frac{^{-}1}{2}$

g. $2.0\overline{312}$

h. 2.0312

i. the infinite sum $\frac{1}{10^1} + \frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^8} + \frac{1}{10^{16}} + \frac{1}{10^{32}} + \frac{1}{10^{64}} + \cdots$

j. π

k. $1432_{(5)}$

l. $1432.2\overline{3100}_{(7)}$

m. $\sqrt{49}$

n. $\sqrt{8}$

o. $0.1411411141111411111411111114\ldots$ (assume the obvious pattern in the digits continues)

p. $0.1411411411411411411411411141\ldots$ (assume the obvious pattern in the digits continues)

52. Simplify the description of each of the following sets.

a. $\mathbb{N} \cap \mathbb{Q}$

b. $\mathbb{N} \cup \mathbb{Q}$

c. $\mathbb{I} \cap \mathbb{Q}$

d. $\mathbb{Q} \cup \mathbb{I}$

e. $\mathbb{I} \cup \mathbb{R}$

f. $\mathbb{Z} \cup \mathbb{Q}$

g. $\mathbb{Z} \cup \mathbb{N}$

h. $\mathbb{R} \cap \mathbb{R}$

i. $\emptyset \cup \mathbb{N}$

j. $\mathbb{N} \cap \emptyset$

k. $\mathbb{R} - \mathbb{Q}$

l. $\mathbb{R} - \mathbb{I}$

53. Which of the following sets is closed under the given operation?

a. The set of even integers under subtraction.

b. The set of even natural numbers under addition.

c. The set of even natural numbers under subtraction.

d. The set of nonzero integers under division.

e. The set of rational numbers under addition.

f. The set of rational numbers under multiplication.

g. The set of nonzero rational nubers under division.

h. The set of irrational numbers under addition.

i. The set of real numbers under square roots.

j. The set of positive real numbers under square roots.

k. The set of integers that are divisible by 6 under addition.

54. State the property of real numbers that justifies the following statements.

a. $\pi + \left(\frac{2}{5} + \sqrt{3}\right) = \left(\frac{2}{5} + \sqrt{3}\right) + \pi$

b. $3\left(\pi\sqrt{2}\right) = \left(\pi\sqrt{2}\right)3$

c. $(3\pi)\sqrt{2} = 3\left(\pi\sqrt{2}\right)$

d. $1 \times \pi = \pi$

e. $\sqrt{2} \times (\pi + 3) = \left(\sqrt{2} \times \pi + \sqrt{2} \times 3\right)$

f. $\sqrt{2} + 0 = \sqrt{2}$

g. $\pi + \left(\frac{2}{5} + \sqrt{3}\right) = \left(\pi + \frac{2}{5}\right) + \sqrt{3}$

55. Simplify the following. Rationalize the denominator where necessary.

a. $\sqrt{2}\sqrt{28}$

b. $\sqrt{2}\sqrt{18}$

c. $\frac{3}{\sqrt{7}}$

d. $\frac{\sqrt{2}}{\sqrt{8}}$

56. Let s be the number of students and p be the number of professors. Write an equation using the variables s and p to represent the following statement: "There are 7 times as many students as professors.".

57. A store sells melons for \$1.20 each and coconuts for \$1.35 each. Let m represent the number of melons sold in one day and c the number of coconuts sold in the same day. Write an algebraic expression that represents each of the following.

a. The total number of melons and coconuts sold.

b. The amount of money obtained from the coconut sales.

c. The total amount of money obtained from the sales of both coconuts and melons on that day.

58. Consider the following three rules for solving linear equations (or inequalities) in a single variable:

Add equals: add (or subtract) the same expression to both sides of the equation
Multiply equals: Multiply (or divide) both sides of the equation the same non-zero expression (for inequalties, multiplying of dividing by a negative number changes the direction of the

inequality)
Simplify: Replace any expression in the statement by an equivalent expression

Using **only these three rules one step at a time** solve the following equations. State which rule you used for each step.

a. $3(x-2) = 4+2x$
b. $3(x-2) < 4+2x$
c. $\frac{3x+1}{x} = 7$
d. $\frac{3}{x} = \frac{4}{x}$ (be careful!)

59. Let x be a real variable. Find the solution set for the following statements.

a. $3x = 2x$
b. $x+1 = x-1$
c. $x = x$
d. $2(x+1) = x+2$
e. $2(x+1) = 2x+2$
f. $x = -x$
g. $(x+1)3-(1+x)3 = 0$
h. $x = x-1$

60. Answer the following.

a. Is $2n$ always greater than $n+2$?
b. If $n-246 = 762$, then what is $n-247$?

61. Do the following.

a. Write an equation whose graph is the line through the points $(^{-}2,6)$ and $(5,^{-}3)$.
b. Write an equation whose graph is the line through the points $(^{-}2,6)$ with slope $^{-}3$.
c. Draw the graph of $f(x) = 2x - \frac{3}{4}$.
d. Compute the slope and y intercept for the graph of $3y - x - 15 = 0$.

62. Do the following.

a. Draw the graph of $f(x) = -x^2 + 2x - 3$
b. Draw the graph of $f(x) = 2^{-x}$

63. If \$5000 is deposited in an interest bearing account which pays continuously compounded interest at an annual rate of 7%, how much money will be in the account after

a. 1 year
b. 5 years
c. 10 years
d. 20 years
e. 50 years

64. A certain amount is deposited initially in an interest bearing account which pays continuously compounded interest at an annual rate of 5.2%. Ten years later the amount in the account has grown to \$1,000,000. How much money was deposited initially?

65. A player rolls a pair of dice. Find the probability of each of the following events.

a. The sum is even.
b. At least one of the die shows an even number.
c. Both of the dice show a prime number.

d. The dice show a pair of consecutive integers (in any order).

e. The values on both die are the same.

f. The values on the dice are not the same.

g. The difference between the two values is three.

66. A lottery works as follows. The player chooses a four digit number. The lottery choosed a four digit number at random. If the player's number matches the lottery's number the play wins $5000. What price should a player be willing to pay for a ticket so that this is a fair game?

67. Five indentical chips labeled A, B, C, D, and E are placed in a box. Two chips are chosen at random. Find the probability of the following events.

a. One of the two chips is B.

b. Neither chip is E.

68. In a survey of 140 fourth graders at Monks Elementary School, 54 students had a dog, 73 students had a cat, and 23 had both a dog and a cat. If a fourth grade student is selected at random, what is the probability that the student will have a dog or a cat?

69. In a standard deck of playing cards, consider the following events:

E: Selecting a face card (jack, queen, or king)
F: Selecting an ace
G: Selecting a spade
H: Selecting a heart

Compute the following probabilities.

a. $P(F)$

b. $P(H)$

c. $P(G \cup H)$

d. $P(E \cup H)$

e. $P(G \cap E)$

70. Given the probabilities, determine the odds in favor of each event.

a. The probability of winning the lottery is $\frac{1}{1{,}000{,}000}$.

b. The probability of selecting a defective part is 4%.

c. There is a 90% chance that the operation will be successful.

71. Given the odds in favor of each event, determine the probability the event.

a. The odds that the Packers will beat the Rams is 4 to 3.

b. The odds of recovering the missing space capsule are 1 to 4.

72. Box A contains two blue marbles and one red marble. Box B contains two white marbles, one red marble, and one blue marble. In a multistage experiment, a marble is first randomly selected from Box A and then a marble is randomly selected from Box B.

a. Draw the probability tree for this experiment.

b. Determine the probability of obtaining at least one blue marble.

c. Determine the probability of obtaining exactly one blue marble.

d. Determine the probability of obtaining two marbles of the same color.

73. What is the probability of getting a sum of 5 at least once on 4 rolls of a pair of dice? (Hint: Use complementary events.)

74. A game consists of rolling a pair of dice, and the number of dollars you receive is equal to the

sum shown on the dice. What is the expected value of this game?

75. State the mathematical definitions of the following terms from geometry. Use complete English sentences and be precise and accurate.

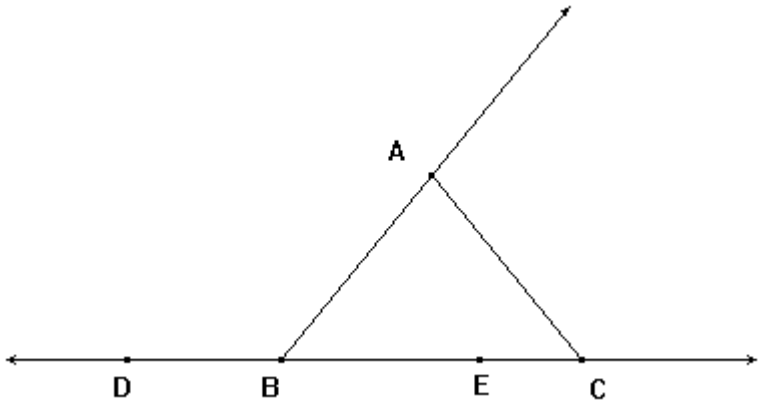
- **a.** ray
- **b.** line segment
- **c.** angle
- **d.** collinear
- **e.** coplanar
- **f.** midpoint
- **g.** straight angle
- **h.** acute angle
- **i.** obtuse angle
- **j.** right angle
- **k.** supplementary angles
- **l.** complementary angles
- **m.** adjacent angles
- **n.** vertical angles
- **o.** alternate interior angles
- **p.** perpendicular lines
- **q.** parallel lines
- **r.** circle
- **s.** radius of a circle
- **t.** diameter of a circle
- **u.** chord of a circle
- **v.** tangent line to a circle
- **w.** polygon
- **x.** triangle
- **y.** quadrilateral
- **z.** pentagon
- **aa.** hexagon
- **bb.** octagon
- **cc.** right triangle
- **dd.** equilateral triangle
- **ee.** isosceles triangle
- **ff.** trapazoid
- **gg.** parallelogram
- **hh.** rhombus
- **ii.** rectangle
- **jj.** square

76. True or False.

- **a.** Every square is a rhombus.
- **b.** Every rhombus is a square.

c. Some parallelograms are rectangles.
d. Every rectangle is a parallelogram.
e. Every equilateral triangle is isosceles.
f. Every trapazoid is a quadrilateral.
g. Some quadrilaterals are trapazoids.

77. In the following diagram, $D, B, E,$ and C are collinear.



a. Name eight line segments that appear in the figure. (For example, $\overline{DA}$ is a line segment, but it does not appear in the figure.)
b. Name two angles (consisting of rays) that appear in the figure. Classify each angle as either obtuse, acute or neither.
c. Name a line that appears in the figure.
d. Name a triangle that appears in the figure.

78. What is the sum of the measures of all the angles in a convex polygon with eleven sides?

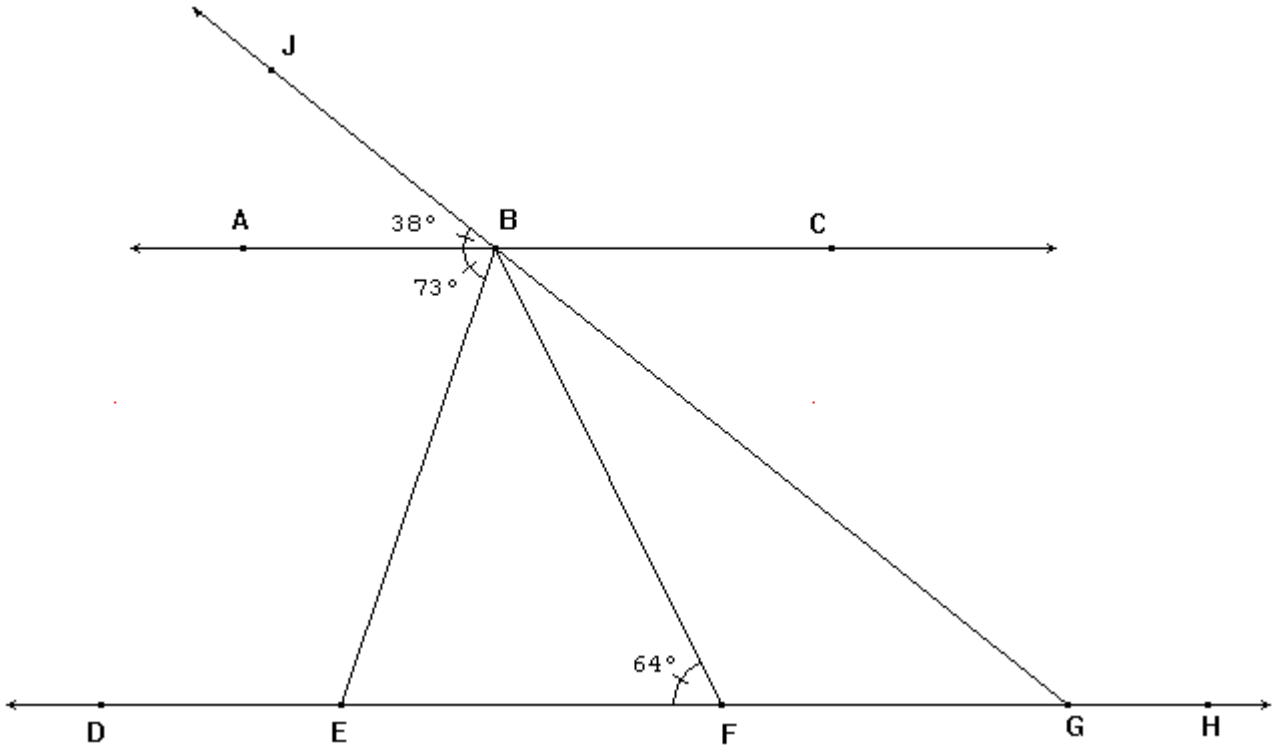
79. What is the measure of each angle in a regular septagon (seven sides)?

80. If the measure of each angle of a regular polygon is 135 degrees, how many sides does the polygon have?

81. Draw a convex quadrilateral and a non-convex quadrilateral.

82. State the alternate interior angle theorem, and the vertical angle theorem.

83. In the following figure $A, B,$ and C are collinear, $J,$ $B,$ and G are collinear, $D,$ $E,$ $F,$ $G,$ and H are collinear, $\overleftrightarrow{AC} \parallel \overleftrightarrow{DH}$, $m\angle JBA = 38°$, $m\angle ABE = 73°$, and $m\angle BFE = 64°$.



Compute the following.

a. $m\angle JBC$

b. $m\angle CBG$

c. $m\angle GBF$

d. $m\angle FBE$

e. $m\angle BED$

f. $m\angle BEF$

g. $m\angle BFG$

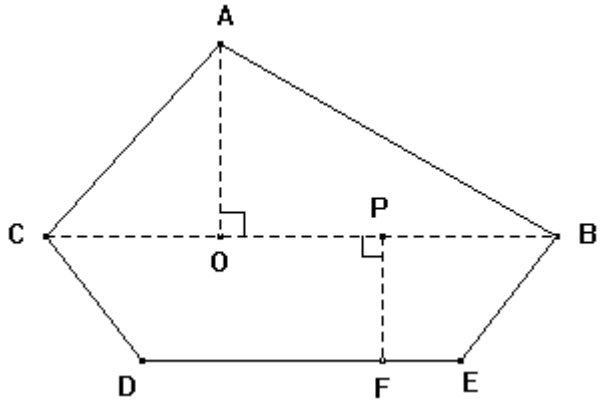
h. $m\angle BGF$

i. $m\angle BGH$

84. Draw a curve that is closed, but not simple.

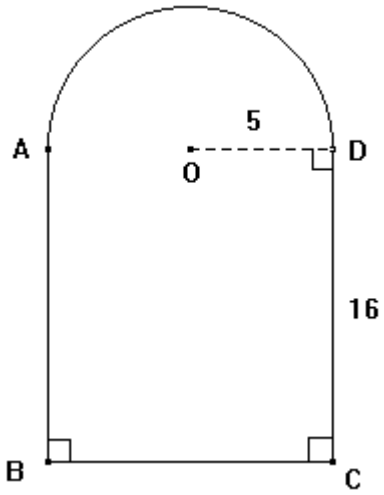
85. Explain why a regular decagon cannot be used to form a regular tessellation of the plane.

86. (In the following problems, XY denotes the length of line segment $\overline{XY}$.) In the figure below, $AO = h$, $BC = r$, $DE = s$, and $PF = k$.



Write a formula for the area of the pentagon $ABEDC$ involving only $h, r, s,$ and k.

87. The figure below consists of half of a circle with center O and radius $\overline{OD}$ on top of a rectangle $ABCD$ with $OD = 5$ and $CD = 16$ as shown.

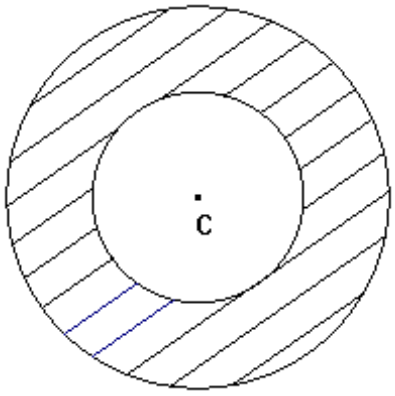


a. What is the area of the entire figure?

b. What is the length of the perimeter of the figure?

88. Let $\triangle ABC$ be a right triangle with right angle at B. If $AB = r$ and $BC = s,$ compute a formula for the area and perimeter of the triangle in terms of r and s.

89. An annulus, is the region enclosed between two concentric circles as shown in the figure:



If the inner circle has center C and a radius of length r and the outer circle has center C and a radius of length s what is the area of the region between the inner and outer circles?

90. An ice cream cone has a radius of 1.5 inches at the top and has a depth (from the center of the top to the tip at the bottom) of 5.2 inches. What is the volume of ice cream that is in the cone, if the ice cream is leveled off to be even with the top of the cone?

91. The Earth is roughly shaped like a sphere with radius of length 6.37×10^6 m. What is the volume and surface area of the Earth?

92. Compute the volume of a right prism whose base is an equilateral triangle of side length s and whose height is h.

93. Compute the volume of cylindrical grain silo of height 60 ft and diameter 7 ft.