Toy Proofs

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The Problem

How to teach undergraduate students to read and write traditional mathematical proofs?
The Transition

Bridging the Gap

Calculation  Formal Calculation  Formal Proof  Proof
Informal Calculation

\[ (x^2 + \sin^3(x))' = (x^2)' + (\sin^3(x))' \]
\[ = 2x + (\sin^3(x))' \]
\[ = 2x + 3 \sin^2(x)(\sin(x))' \]
\[ = 2x + 3 \sin^2(x) \cos(x) \]
(x^2 + \sin^3(x))' = (x^2)' + (\sin^3(x))' \\
= 2x + (\sin^3(x))' \\
= 2x + 3 \sin^2(x)(\sin(x))' \\
= 2x + 3 \sin^2(x) \cos(x) \\
\text{by the sum rule} \\
\text{by the power rule} \\
\text{by the chain rule} \\
\text{by the derivative of sin rule}
Calculation or Proof?

**Theorem:** \((x^2 + \sin^3(x))' = 2x + 3\sin^2(x)\cos(x)\).

**Proof:**

\[
(x^2 + \sin^3(x))' = (x^2)' + (\sin^3(x))' \\
= 2x + (\sin^3(x))' \\
= 2x + 3\sin^2(x)(\sin(x))' \\
= 2x + 3\sin^2(x)\cos(x)
\]

by the sum rule

by the power rule

by the chain rule

by the derivative of sin rule.
Formal Proof to Proof

Proof: Let $\angle BAC'$ be an angle. There exists a point $C$ on $\overrightarrow{AC'}$ with $AB \equiv AC$ and $\triangle ABC$ is isosceles, so $\angle ABC \equiv \angle ACB$. Hence $BM \equiv MC$ and $\triangle AMB \equiv \triangle AMC$ by the point plotting theorem, by the definition of isosceles, by the isosceles triangle theorem, by SMSG Thm 10, by the definition of midpoint, by the SAS axiom (S11). So ...
The Formal Proof Game

1. Statements or Expressions (*Toys!*)
   The language used in the particular proof or calculation.

2. The Goal (*How to Win!*)
   The statement we are trying to prove.

3. Hypotheses (*The Starting Position!*)
   Statements we are given initially.

4. Rules of Inference (*Rules of the Game!*)
   Allow us to construct new statements from ones we already have.
Proof Parts

Goal

Theorem: \( (x^2 + \sin^3(x))' = 2x + 3\sin^2(x)\cos(x) \).

Proof:

\[
(x^2 + \sin^3(x))' = (x^2)' + (\sin^3(x))' \\
= 2x + (\sin^3(x))' \\
= 2x + 3\sin^2(x)(\sin(x))' \\
= 2x + 3\sin^2(x)\cos(x)
\]

by the sum rule
by the power rule
by the chain rule
by the derivative of sin rule.

Statements

Rules
Toy Proofs

- Introductory Proof-like Games
  - Scrambler
  - TriX
  - Circle-Dot
Circle-Dot

- **Statements**: any sequence of circles and dots
- **Rules** (for any statements \(W\) and \(V\))
  
  **Axiom 1**: \(\bigcirc\bigcirc\)
  
  **Axiom 2**: \(\bullet\bigcirc\)

**Rule 1**: Given \(WW\) and \(VV\), conclude \(W\)

**Rule 2**: Given \(W\) and \(V\), conclude \(W\bullet V\)

**Rule 3**: Given \(WV\bullet\), conclude \(W\bigcirc\)
The Lurch Project

- **Lurch: Software for Teaching Mathematical Proofs**
  National Science Foundation Grant No. 0736644
  - Like a spell-checker or grammar-checker, but for mathematical reasoning.
  - **Mission Statement**: *Lurch should be as indistinguishable from the ordinary activities of mathematics as possible, except for the additional services it provides.*
  - Support for many common undergraduate topics planned: algebra, trigonometry, calculus, logic, set theory, number theory, group theory, etc.
For more info...

The Toy Proof software and these slides are available at the Lurch project home page:

http://lurch.sourceforge.net/