Tarski's Axioms

Undefined terms: *between* and *congruent*

Notation:

- $ab \equiv cd$ is an abbreviation for *congruent(a,b,c,d)*
- $a.b.c$ is an abbreviation for *between(a,b,c)*

Note: We can think of all variables as representing points, $ab \equiv cd$ as saying "the distance from point a to point b is the same as the distance from point c to point d ", and $a.b.c$ as saying "b is between a and c"... but we don't have to! All free variables in the following axioms are assumed to be quantified by $\forall$.

T1: $ab \equiv ba$
T2: $ab \equiv pq$ and $ab \equiv rs \Rightarrow pq \equiv rs$
T3: $ab \equiv cc \Rightarrow a = b$
T4: $\exists x, q.a.x$ and $ax \equiv bc$
T5: ($a \neq b$ and $a.b.c$ and $a'.b'.c'$ and
$ab \equiv a'b'$ and $bd \equiv b'd'$ and $bc \equiv b'c'$ and $ad \equiv a'd'$) $\Rightarrow cd \equiv c'd'$
T6: $a.b.a \Rightarrow a = b$
T7: $a.p.c$ and $b.q.c \Rightarrow \exists x, p.x.b$ and $q.x.a$
T8: $\exists a \exists b \exists c, \sim\big(a.b.c$ or $a.c.b$ or $c.a.b\big)$
T9: $p \neq q$ and $ap \equiv aq$ and $bp \equiv bq$ and $cp \equiv cq \Rightarrow a.b.c$ or $a.c.b$ or $b.a.c$
T10: $a.d.t$ and $b.d.c$ and $a \neq b \Rightarrow \exists x \exists y, a.b.x$ and $a.c.y$ and $x.t.y$
T11: $\big(\exists a \forall x \forall y, \big(\Phi(x)$ and $\Psi(y)\big) \Rightarrow a.x.y\big) \Rightarrow \big(\exists b \forall x \forall y, \big(\Phi(x)$ and $\Psi(y)\big) \Rightarrow x.b.y\big)$

Note: T11 is actually an axiom scheme, one axiom for each formula Φ and Ψ.

[Source: Szczerba, *Tarski and Geometry*, Journal of Symbolic Logic, 51, **4** (1986), 907-912]