Assignment 5 (additional problems)

1. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $f(x,y) = xy$. Prove f is surjective.

2. Let $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ by $f(n) = (n,n)$. Prove f is injective.

3. Let $f : A \to B$. Define $g : A \to f(A)$ by $\forall x \in A, g(x) = f(x)$. Prove g is surjective.

[Comment from your fearless leader: The only difference between the maps g and f is that they have different codomains, with the codomain of g being the range of f. This problem shows that any function f can be thought of as “onto its range”, even if f itself is not onto.]