Section 2.1: Congruence in $\mathbb{Z}$

In the following recipes, $n \in \mathbb{N}^{+}$, $a,b \in \mathbb{Z}$, and $[\ \ ]$ is $[\ \ ]_{\underset{n}{\equiv}}$.

Recipes for Section 2.1	
$\underset{n}{\equiv}$ $+$ 1. Show $n \mid a - b$ Conclude $a \underset{n}{\equiv} b$	$\underset{n}{\equiv}$ $-$ 1. Show $a \underset{n}{\equiv} b$ Conclude $n \mid a - b$
Thm 1 Conclude $\underset{n}{\equiv}$ is an equiv reln	
$\mathbb{Z}_n$ $+$ 1. Show $x = [a]$ Conclude $x \in \mathbb{Z}_n$	$\mathbb{Z}_n$ $-$ 1. Show $x \in \mathbb{Z}_n$ Conclude $x = [a]$ for some $a \in \mathbb{Z}$
Cor to Burning Thm 1. Show $[a] = [b]$ Conclude $a \underset{n}{\equiv} b$	Cor to Burning Thm 1. Show $a \underset{n}{\equiv} b$ Conclude $[a] = [b]$
Cor 2a 1. Show $q,r \in \mathbb{Z}$ 2. Show $a = nq + r$ 3. Show $0 \leq r < n$ Conclude $[a] = [r]$	
Cor 2b 1. Show $x \in \mathbb{Z}_n$ Conclude $x \in \{[0],[1],\ldots,[n-1]\}$	Cor 2b 1. Show $i,j \in \{0,1,\ldots,n-1\}$ 2. Show $[i] = [j]$ Conclude $i = j$

Section 2.2: Arithmetic in $\mathbb{Z}_n$

In the following recipes let $n \in \mathbb{N}^+, a, b, c, d \in \mathbb{Z}$ and $[\ \ ]$ denote $[\ \ ]_{\underset{n}{\equiv}}$.

Recipes for Section 2.2	
Sec 2.2 Thm 1 1. Show $a \underset{n}{\equiv} b$ 2. Show $c \underset{n}{\equiv} d$ Conclude $a + c \underset{n}{\equiv} b + d$ Conclude $ac \underset{n}{\equiv} bd$	
$\mathrm{Mod}\ +$ 1. Show $q, r \in \mathbb{Z}$ 2. Show $a = nq + r$ 3. Show $0 \leq r < n$ Conclude $r = \left(a \ \mathrm{Mod}\ n\right)$	**$\mathrm{Mod}\ -$** 1. Show $r = a \ \mathrm{Mod}\ n$ Conclude $r \in \mathbb{Z}$ Conclude $a = nq + r$ for some $q \in \mathbb{Z}$ Conclude $0 \leq r < n$
binary operator $+$ 1. Show $f : X \times X \to X$ Conclude f is a binary operator on X	**binary operator $-$** 1. Show f is a binary operator on X Conclude $f : X \times X \to X$
Sec 2.2 Thm 2 Conclude $\oplus : \mathbb{Z}_n \to \mathbb{Z}_n$ Conclude $\otimes : \mathbb{Z}_n \to \mathbb{Z}_n$	
Def $\oplus$ Conclude $[a] \oplus [b] = [a + b]$	**Def $\otimes$** Conclude $[a] \otimes [b] = [ab]$
Sec 2.2 Thm 3 1. Show $A, B, C \in \mathbb{Z}_n$ Conclude $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ Conclude $A \oplus B = B \oplus A$ Conclude $[0] \oplus A = A \oplus [0] = A$ Conclude $\exists X \in \mathbb{Z}_n, A \oplus X = [0]$ Conclude $A \otimes (B \otimes C) = (A \otimes B) \otimes C$ Conclude $A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C)$ Conclude $A \otimes B = B \otimes A$ Conclude $A \otimes [1] = [1] \otimes A = A$	**Mult by 0 in $\mathbb{Z}_n$** 1. Show $A \in \mathbb{Z}_n$ Conclude $[0] \otimes A = [0]$