

Homework assignment: Modern Algebra II  
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Let  $G$  be the subset of  $M_4(\mathbb{Z}_2)$  consisting of those  $4 \times 4$  matrices having exactly one 1 in each row and in each column. Define the product  $*$  on  $G$  to be matrix multiplication (not addition!).

1. Prove  $(G, *)$  is a group.
2. Compute  $|G|$ .
3. Is  $G$  Abelian? Prove or disprove.
4. Is  $G$  isomorphic to  $Z_{24}$ ? Prove or disprove.
5. Is  $G$  isomorphic to  $S_4$ ? Prove or disprove.
6. Is  $G$  isomorphic to  $D_{12}$ ? Prove or disprove.
7. Is  $G$  isomorphic to  $\langle a \rangle$  for any matrix  $a$ ? Prove or disprove.
8. Find the orders of all of the elements of  $G$ .
9. Find the conjugacy classes of  $G$ .
10. Find all of the centralizers of the elements of  $G$ .
11. Find the center of  $G$ .
12. Find all of the Sylow 2-subgroups of  $G$ .
13. Find all of the Sylow 3-subgroups of  $G$ .
14. Find the normalizers of the Sylow  $p$ -subgroups you found in #12 and #13.
15. Write the class equation for  $G$ .